
**Set:** any collection of elements.

**Elements:** objects of the set.

\( \in \) means “belongs to” or “is an element of”

\( \notin \) means “is not an element of”

**Roster Method**

**Exercise 1.** The set \( S \) of names of states beginning with letter \( A \).

Natural Numbers
Exercise 2. (You Try!) Use the roster method to do the following:

a) The set of even natural numbers from 80 to 90.

b) The set of odd natural numbers greater than 10.

Exercise 3. (You Try!) Decide whether each statement is true or false.

a) \(27 \in \{1, 5, 9, 13, 17, \ldots \}\)

b) \(z \in \{v, w, x, y, z\}\)

c) \( \text{map} \notin \{m, a, p\}\)

The Descriptive Method

Exercise 4. Use the descriptive method to describe the set \(B\) containing 2, 4, 6, 8, 10, and 12.

Exercise 5. (You Try!) Use the descriptive method to describe the set \(A\) containing -3, -2, -1, 0, 1, 2, 3.
Set-builder notation

Exercise 6. The set \(\{1, 2, 3, 4, 5, 6\}\) can be written in set-builder notation as 
\(\{x| x \in N \text{ and } x < 7\}\)

Exercise 7. Use set-builder notation to designate each set, then write how your answer would 
be read aloud.

a) The set \(R\) contains the elements 2, 4, and 6.

b) The set \(W\) contains the elements red, yellow, and blue.

Exercise 8. (You Try!) Use set-builder notation to designate each set, then write how your 
answer would be read aloud.

a) The set \(K\) contains the elements 10, 12, 14, 16, 18.

b) The set \(W\) contains the elements Democrat and Republican.

Exercise 9. Designate the set \(S\) with elements 32, 33, 34, 35, \ldots using 

a) The roster method
Exercise 10. (You Try!) Designate the set $E$ with elements 11, 13, 15, 17, ... using
a) The roster method

b) The descriptive method.

c) Set-builder notation.

Writing a Set Using Ellipsis

Exercise 11. Using the roster method, write the set containing all even natural numbers between 99 and 201.

Exercise 12. (You Try!) Using the roster method, write the set of odd natural numbers between 50 and 500.
**Empty set (or null set):** set which contains no elements and is written $\emptyset$ or {}.

**Identifying Empty Sets**

**Exercise 13.** Which of the following sets are empty?

a) $\{\emptyset\}$

b) $\{x | x$ is a natural number between 1 and 2\}$

c) $\{x | x$ is a human being living on Mars\}$

**Cardinal Number of a Set**

**Definition 1.** The number of elements in a set is called the **cardinal number** of a set. For a set $A$ the symbol for the cardinality is $n(A)$, which is read as “n of A.”

**Exercise 14.** If $R = \{2, 4, 6, 8, 10\}$, then one could say that the **cardinality** of set $R$ is 5 or write $n(R) = 5$.

**Exercise 15.** Find the cardinal number of each set.

a) $A = \{5, 10, 15, 20, 25, 30\}$

b) $B = \{x | x \in N$ and $x < 16\}$

c) $\emptyset$

d) $A = \{z, y, x, w, v\}$

e) $C = \{Chevrolet\}$
Finite and Infinite Sets

**Definition 2.** A set is called **finite** if it has no elements, or has cardinality that is a natural number. A set that is not finite is called an **infinite set**.

**Exercise 16.** Classify each set as finite or infinite.

a) \{x|x \in N \text{ and } x < 100\}

b) \{100, 102, 104, 106, \ldots \}

c) Set M is the set of people in your immediate family.

d) Set S is the set of songs that can be written.

Equal and Equivalent Sets

**Definition 3.** Two sets A and B are **equal** (written \(A = B\)) if they have exactly the same members or elements. Two finite sets A and B are said to be **equivalent** (written \(A \cong B\)) if they have the same number of elements: that is, \(n(A) = n(B)\)

a) \{a, b, c\} and \{c, b, a\} are equal.

b) \{4, 5, 6\} and \{4, 4, 5, 6\} are equal (4 need not be written twice in the second set).

c) the set of all names of students in your class and the set of their ID numbers are equivalent sets because they have the same number of elements, but they have different elements so the sets are not equal.
Exercise 17. State whether each pair of sets is equal, equivalent, or neither.

a) \{p, q, r, s\}; \{a, b, c, d\}

b) \{2, 13\}; \{2, 1, 3\}

c) \{ even natural numbers less than 10 \}; \{2, 4, 6, 10\}

d) \{t, o, p\}; \{p, o, t\}

Definition 4. Two sets have a **one-to-one correspondence** of elements if each element in the first set can be paired with exactly one element of the second set and each element of the second set can be paired with exactly one element of the first set.

Exercise 18. Show that

a) the sets \{8, 16, 24, 32\} and \{s, t, u, v\} have a one-to-one correspondence.

b) the sets \{x, y, z\} and \{5, 10\} do not have a one-to-one correspondence.
Section 2.2: Subsets and Set Operations

Suppose that we are given a set $A$. We may form new sets by selecting elements from $A$. Sets formed in this way are called subsets of $A$.

Empty set (or null set): set which contains no elements and is written $\emptyset$.

Note: Empty set is a subset of every set.

Example 5. Let $A = \{a, b, c\}$. Find all subsets of $A$.

If a set $A$ is a subset of a set $B$ and is not equal to $B$, then we call $A$ a proper subset of $B$, and write $A \subset B$.

Example 6. Find all proper subsets of $\{\emptysuit, \heartsuit, \spadesuit, \clubsuit\}$
Note: If a finite set has \( n \) elements, then the set has \( 2^n \) subsets and \( 2^n - 1 \) proper subsets.

**Universal Set:** denoted by \( U \) is the set containing all sets being discussed.

**Complement of \( A \):** denoted by \( A' \), is the set of elements of \( U \) which are not in \( A \).

**Set theoretic notation:**

*Example 7.* \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) and \( A = \{2, 4, 6, 8\} \). Find \( A' \).

Let \( A \) and \( B \) be sets.

1. The set \( A \cup B \), called the **union** of \( A \) and \( B \), consists of all elements which are in \( A \) or in \( B \) or in both.

2. The set \( A \cap B \), called the **intersection** of \( A \) and \( B \), consists of all elements which are in both \( A \) and \( B \).

**Exercise 19.** Let \( U = \{1, 2, 3, 4, 5, 6, 7\} \), \( S = \{1, 2, 3, 4\} \), and \( T = \{1, 3, 5, 7\} \). List the elements of the sets
   a) \( S' \)
   b) \( S \cup T \)
   c) \( S \cap T \)
   d) \( S' \cap T \)

**Exercise 20.** Let \( U = \{a, b, c, d, e, f, g\} \), \( R = \{a, b, c, d\} \), \( S = \{c, d, e\} \), and \( T = \{c, e, g\} \). List the elements of the sets
Exercise 21. Let $U = \{10, 20, 30, 40, 50, 60, 70, 80\}$, $A = \{10, 30, 50, 70\}$, $B = \{40, 50, 60, 70\}$, and $C = \{20, 40, 60\}$. List the elements of the sets

a) $A' \cap C'$

b) $(A \cap B)' \cap C$

c) $B' \cup (A \cap C')$

Finding the Difference of Two Sets

Exercise 22. Let $A = \{4, 6, 8, 10\}$, $B = \{2, 6, 12\}$, and $C = \{8, 10\}$. Find each set.

a) $A - B$

b) $B - C$

c) $(A - B) - C$

Finding Cartesian Products

The Cartesian product (denoted $A \times B$) of two sets $A$ and $B$ is formed by writing all possible ordered pairs in which the first component is an element of $A$ and the second component is an element of $B$.

Set-builder notation: $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$. 
Exercise 23. If $A = \{\text{freshman, sophomore, junior}\}$ and $B = \{\text{quarterback, running back}\}$, find $A \times B$ and describe its significance.

Section 2.3: Using Venn Diagrams to Study Set Operations

Drawing a Venn Diagrams

a) $A$

b) $A'$

c) $A \cup B$

d) $A \cap B$
e) $A \cap B'$

f) $A' \cap B'$

g) $(A \cup B)'$

h) $A' \cup B'$

i) $A \cap B \cap C$
j) \( A \cap B \)

k) \( A \cap (B \cup C) \)

l) \( (A \cap B) \cup C \)

m) \( (A \cap B') \cup C \)
Exercise 24. Write the set illustrated by the Venn diagram below.

De Morgan’s Laws

For any two sets $A$ and $B$,

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Using Sets to Solve Problems

**Inclusion-Exclusion Principle** Let $S$ and $T$ be set. Then

$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

**Note:** If $S \cap T = \emptyset$, then $n(S \cup T) =$
Exercise 25. Find \( n(S \cup T) \), given that \( n(S) = 5 \), \( n(T) = 4 \), and \( n(S \cap T) = 2 \).

Note: Keep in mind that “OR” means \( \cup \) and “AND” means \( \cap \).

Exercise 26. Suppose that each of the 245 million adults in South America is fluent in Portuguese or Spanish. If 134 million are fluent in Portuguese and 130 million are fluent in Spanish, how many are fluent in both languages?

Exercise 27. In the year 2007, *Executive* magazine surveyed the presidents of the 500 largest corporations in the United States. Of these 500 people, 310 had degrees (of any sort) in business, 238 had undergraduate degrees in business, and 184 had graduate degrees in business. How many presidents had both undergraduate and graduate degrees in business?
Exercise 28. Motors Inc. manufactured 325 cars with automatic transmissions, 216 with power steering, and 89 with both of these options. How many cars were manufactured with at least one of the two options?

Exercise 29. Draw an appropriate Venn diagram and use the given data to determine the number of elements in each basic region.

a) \( n(U) = 20, n(S) = 12, n(T) = 14, n(S \cup T) = 18 \)

b) \( n(U) = 75, n(S) = 15, n(T) = 25, n(S^c \cap T^c) = 40 \)
c) \( n(R') = 22, n(R \cup S) = 21, n(S) = 14, n(T) = 22, n(R \cap S) = 7, n(S \cap T) = 9, n(R \cap T) = 11, n(R \cap S \cap T) = 5 \)
Exercise 30. An advertising agency finds that the media use of its 170 clients is as follows:

115 use television (T)
100 use radio (R)
130 use magazines (M)
75 use television and radio
95 use radio and magazines
85 use television and magazines
70 use all three.

a) Complete the Venn diagram.

b) How many people use radio only?
c) How many people use all three of the media?
d) How many people use magazines but not television?
e) How many people use exactly one of the three media?
f) How many people use at least one of the three media?
g) How many people use none of the three media?
h) How many people use exactly two of the three media?
i) How many people use at least two of the three media?
j) How many people use radio or magazines?
k) How many people use television and radio?
l) How many people use television and radio, but not magazines?
Exercise 31. (You Try!) Three of the most dangerous risk factors for heart attack are high blood pressure, high cholesterol, and smoking. In a survey of 690 heart attack survivors, 62 had only high cholesterol among those three risk factors; 36 had only smoking, and 93 had only high blood pressure. There were 370 total with high cholesterol, 159 with high blood pressure and cholesterol that didn’t smoke, and 23 that smoked and had high cholesterol but not high blood pressure. Finally, 585 had at least one risk factor. Draw a Venn diagram representing this information and use it to answer the following questions.

a) How many survivors had all three risk factors?

b) How many had exactly two of the three risk factors?

c) How many had none?

d) What percentage were smokers?