Applications of the Normal Distribution

Diana Pell

Exercise 1. Three red cards are numbered 1, 2, and 3. Three black cards are numbered 4, 5, and 6. The cards are placed in a box and one card is picked at random.

a) Find the probability that a red card was picked given that the number on the card was an odd number.

b) Find the probability that a number less than 5 was picked given that the card was a black card.

c) Find the probability that a number less than 5 was picked given that the card was red.

d) Find the probability that a black card was picked given that the number on the card was an even number.

Exercise 2. A survey of 200 college students shows the average number of minutes that people talk on their cell phones each month.

<table>
<thead>
<tr>
<th></th>
<th>Less than 600</th>
<th>600-799</th>
<th>800-999</th>
<th>1,000 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>56</td>
<td>18</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>Women</td>
<td>61</td>
<td>18</td>
<td>13</td>
<td>8</td>
</tr>
</tbody>
</table>

If a person is selected at random, find these probabilities:

1) The student talked less than 600 minutes given that it was a woman.

2) The student talked more than 999 minutes given that it was a man.

3) The student was a woman given that they used between 600 and 799 minutes.

4) The student was a man given that they used between 600 and 999 minutes.

A standard package of Oreos is supposed to contain 510 grams of chocolatey goodness. But there’s variation in just about anything, including production and packaging, so some packages will contain more and some will contain less. In fact, this is exactly the sort of quantity that tends to be normally distributed. The folks that run Nabisco aren’t stupid, and they know that customers won’t be very happy if they weigh a package of cookies and find that it contains less than the labeled amount. The typical approach to keep that from happening is to design the packaging process so that the mean is something more than 510 grams, with a standard deviation that guarantees that the vast majority of packages contain 510 grams or more.
**Exercise 3.** Let's say that the mean is 518 grams and the standard deviation is 4 grams.

a) Fill in all of the blanks on the empirical rule diagram below with weights in grams. Use the formulas below the blanks for reference.

![Empirical Rule Diagram](Image)

**Figure 1:**

b) What percent of all Oreo packages would contain between 514 and 522 grams?

c) Using what we know about the connection between probability and percent chance, what’s the probability that a randomly chosen package contains between 514 and 522 grams?

d) Find the probability that a randomly chosen package contains less than 510 grams of cookies?

e) Based on your answer to part (d), if 1,000 Oreo packages are sampled, how many will have less than 510 grams of cookies?

**Some Properties of a Normal Distribution**

1) It is bell-shaped.

2) The mean, median, and mode are all exactly the same, and are located at the center of the distribution.

3) It’s symmetric about its mean.

4) The total area under the entire curve is 1.

For a given data value from a data set that is normally distributed, we define that value’s z-score to be

\[ z = \frac{\text{Data value} - \text{mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma} \]
**Exercise 4.** If the weights of Oreos in a package are normally distributed with mean 518 grams and standard deviation 4 grams, find the percentage of packages that will weigh less than 510 grams.

**Exercise 5.** Based on data compiled by the World Health Organization, the mean systolic blood pressure in the United States is 120, the standard deviation is 16, and the pressures are normally distributed. Find each.

a) The percent of individuals who have a blood pressure between 120 and 128

b) The percent of individuals who have a blood pressure above 132

c) The percent of individuals who have a blood pressure between 112 and 116

d) The percent of individuals who have a blood pressure between 124 and 144

e) The percent of individuals who have a blood pressure lower than 104

**Exercise 6.** Based on data in the 2012 Statistical Abstract of the United States, the average American generates 1,570 pounds of garbage per year. Let’s estimate that the number of pounds generated per person is approximately normally distributed with standard deviation 200 pounds. Find the probability that a randomly selected person generates

a) Between 1,250 and 2,050 pounds of garbage per year.

b) More than 2,050 pounds of garbage per year.

**Exercise 7.** The Statistical Abstract also indicates that of the 1,570 pounds of garbage generated by the average individual, 872 pounds will end up in a landfill. If these amounts are approximately normally distributed with standard deviation 160 pounds, find the probability that a randomly selected person generates

a) Less than 600 pounds that end up in a landfill.

b) Between 600 and 1,000 pounds that end up in a landfill.

**Exercise 8.** The mean for a reading test given nationwide is 80, and the standard deviation is 8. The random variable is normally distributed. If 10,000 students take the test, find each.

a) The number of students who will score above 90

b) The number of students who will score between 78 and 88

c) The number of students who will score below 76

**Exercise 9.** If the mean salary of public school teachers in the United States in 2012 was $50,547 and the standard deviation was $9,000, find these probabilities for a randomly selected teacher. Assume the variable is normally distributed.

a) The teacher earns more than $55,000.

b) The teacher earns less than $45,000.
Exercise 10. A survey found that people keep their televisions an average of 4.8 years. The standard deviation is 0.89 year. If a person decides to buy a new TV, find the probability that he or she has owned the old one for the given amount of time. Assume the random variable is normally distributed.

a) Less than 2.5 years
b) Between 3 and 4 years
c) More than 4.2 years

Exercise 11. The average age of CEOs is 56 years. Assume the random variable is normally distributed. If the standard deviation is 4 years, find the probability that the age of a randomly selected CEO will be in the given range.

a) Between 53 and 59 years old
b) Between 58 and 63 years old

c) Between 131 and 136