Webassign 3, Problems 8, 9, 10: Relative Velocity

These three problems are the only place you will see the topic of relative velocity this semester. Much like the last two Webassign problems from Chapter 1, in which you worked with displacement in three dimensions, I only include these problems so that you have some exposure to the topic. Relative velocity does not play a role in anything else we will do this semester.

As I wrote in the Chapter 3 Notes, relative velocity problems involve three velocities:

- $v_1$ is the object’s velocity within its “frame of reference”
- $v_2$ is the velocity of the “frame of reference”
- $v_3$ is the velocity of the object relative to anything (or anyone) outside of the “frame of reference”

I gave the simple example in class of me walking on a train. The train would be my “frame of reference”. If I walked at a speed of 3 mph (i.e. $v_1$) and the train moved in the same direction at a speed of 10 mph (i.e. $v_2$) then people outside of the train would see me move at a speed of 13 mph (i.e. $v_3$).

Simple principle with relative velocities: $v_3 = v_1 + v_2$

In each of these problems we simply need to identify $v_1$, $v_2$, and $v_3$. Some problems are two dimensional, so we will need to use the above equation for the x direction and y direction separately. We will make triangles in some problems.

Problem 8
In this problem, the swimming speed of the student in still water is given. This is $v_1$. The swimmer’s “frame of reference” is the water, so the speed of the water is $v_2$. The swimmer’s speed relative to someone on land is $v_3$.

There are two parts to this problem: the swimmer swims with the current one way and swims against the current the other way. You can find $v_3$ in each case:

- swims with the current: $v_3 = v_1 + v_2$ (note that this is just like my train example.)
- swims against the current: $v_3 = v_1 - v_2$

In each case, get $v_3$ and use it to determine the time to swim the given distance. Add the two times to get the total time for the roundtrip.

For the second part, repeat the first part with $v_2 = 0$. Of course this means that $v_3$ is the same as $v_1$, which makes perfect sense: if the water is not moving, then the speed of the swimmer in the water is the speed of the swimmer relative to the shore.

Problem 9
This one is a little sneaky, but we can pull it apart and figure it out. The key to this problem is to realize that you are in the car watching the rain fall outside. The rain is falling straight down, vertically, relative to the ground. Your car is moving horizontally.
A simple principle of motion (which Einstein used to develop his Theory of Special Relativity) is that all motion is relative. When we are in a moving car, we know that the ground is sitting still and we are moving. But what we observe outside looks exactly the same as if we were sitting still and the world around us was moving past us! This is an important principle and we will take advantage of it in this problem.

Imagine that you are in the car in the problem. The problem states that you (and the car) are moving at a given speed. But this means from our perspective the world outside appears to move past us at that speed. Now we can assign:

velocity of rain in the world outside: \( v_1 = ? \) (we know that \( v_1 \) is vertical... rain falls straight down)
velocity of the world outside (the rain’s “frame of reference”): \( v_2 \) is given and is horizontal
velocity you observe of the rain: \( v_3 \)

Draw a triangle of these three velocities: \( v_1 \) is vertical, \( v_2 \) is horizontal, and \( v_3 \) is the hypotenuse, because it is the combination of \( v_1 \) and \( v_2 \) added together.

The answer to part (a) is the magnitude of \( v_3 \), the hypotenuse of the triangle. The answer to part (b) is the magnitude of \( v_1 \), the vertical side of the triangle.

**Problem 10**

In this problem, the velocity of the airplane in the air is \( v_1 \), since the air is the airplane’s “frame of reference” (just as the swimmer swims in the water while the water moves, in this problem the airplane flies through the air while the air is moving, relative to the ground.)

The velocity of the “wind” is \( v_2 \), since the wind is the motion of the air, which is the airplane’s “frame of reference.” Note how, once again, this is the same as my “walking on the train” example.

If you assign “north” to the y-axis and “west” to the x-axis, then you are given \( v_1 \) is in the x-direction and \( v_2 \) is in the y-direction.

Start from the origin; draw an arrow in the x-direction representing \( v_1 \). From the end of that arrow, draw another arrow in the y-direction representing \( v_2 \). Having just drawn two legs of a triangle, draw the hypotenuse. The hypotenuse represents the “total velocity”, i.e. the magnitude of \( v_3 \), which is \( v_1 \) and \( v_2 \) added together.

Notice that all units are given in km/h and you are asked for \( v_3 \) in km/h. Please do not convert units, unless you are just doing it for fun because you can’t get enough practice converting units.

The direction of \( v_3 \)? Since “west” is the x-direction, you need the angle between the hypotenuse and the x-axis.