These notes are eight pages. That includes some diagrams, but I realize reading them could get a bit tedious. So here is a quick summary:

- A “vector quantity” is one for which direction is relevant, like velocity, acceleration or force.
- When working with a vector quantity, it is important to always consider direction.
- We graphically represent a vector quantity with an arrow pointing in the correct direction.
- The length of the arrow is an expression of the magnitude of the quantity.
- Every arrow representing a vector quantity is the hypotenuse of a right triangle.
- The sides of the triangle are called the two “components” of the vector.
- The lengths of the sides can be determined using basic trigonometry.

**What is a “Vector Quantity”?**

Every definition we make this semester will fall into one of two categories: scalar quantity or vector quantity. A vector quantity is one for which direction must be considered; a scalar quantity is one for which direction does not matter or is nonsensible.

The “test” for this classification will be simple: when we make a definition, we will simply ask “what direction is it?” If that question makes sense, we know that definition is a vector quantity. If it does not make sense, the definition is a scalar quantity. For example, I could express the velocity of my car by claiming “I drove 75 mph this morning.” You could ask “What direction?” This question makes sense! We classify “velocity” as a vector quantity. *Most of our definitions of motion will be vector quantities, because the description of motion inherently involves direction.*

In contrast, I could tell you “The mass of my book is 1.3 kg.” If you were to ask “What direction is it?” an uncomfortable silence would follow... because that question makes no sense. We know that mass is measurable and quantifiable, but there is no direction associated with it. Mass is a scalar quantity.

A few more examples:

- **Scalar quantities:** time, mass, temperature, energy
- **Vector quantities:** displacement, velocity, acceleration, force, momentum

We will encounter many vector quantities this semester because this semester is entirely about the study of motion. Most concepts that describe motion involve direction.

**How Do We Work With Vector Quantities?**

When measuring or expressing a vector quantity, we must always include the direction of the quantity. For this reason, vector quantities have two pieces of information: their magnitude and their...
direction. The “magnitude” of a vector quantity is simply the quantifiable amount. For example, the magnitude of a velocity might be 30.5 m/s or 75 mph. “Magnitude” might seem like an odd way of expressing this idea of “quantity”, but keep in mind that someone chose this vocabulary and everyone else went along with it. Which means you have to also. Don’t overthink it, just go with the crowd.

Our first vector quantity is displacement. The displacement of an object is defined as the distance and direction the object moved from its initial location to its final location. Displacement is related to “distance” but it is different: while a measure of distance will represent how far an object moves, its displacement expresses the final location of the object (relative to the initial location.)

The direction of a vector is usually expressed by comparison to an established “reference direction”. If we are working with directions on the surface of the Earth, we might use a cardinal direction (north, south, east or west) as a reference. For example, you might express a velocity as:

$$26.8 \text{ m/s } 30^\circ \text{ north of east}$$

Note that this manner of expression includes the magnitude (26.8 m/s) and the direction (30° north of east).

If you are asked to calculate a quantity that is a vector quantity, you must always include the direction in your answer!

Another way we can express a vector is best illustrated by considering a “displacement” vector. Displacement is simply the distance from an initial location to a final location, and the direction. To illustrate a displacement, we simply draw a line from Point A to Point B. The length of that line is the magnitude of the displacement; we express the direction of the displacement by considering the angle the displacement vector makes with a reference direction. For example:
Here I have identified a vector, which I have called “D”, which represents the displacement from point A to point B. Note that the arrow on the line representing the vector points toward “B”. The magnitude of vector D is 14.6 meters and the direction is 28.3 degrees “north of east”.

Using north-south-east-west as reference directions works very well if we are only interested in displacement or velocity vectors which refer to motion on the surface of the Earth. But we need a system which allows us to express vectors without restriction. The solution is simple: we use the classic “x and y” directions of a Cartesian coordinate system from basic algebra. Using this system, our vector above might be:

![Diagram of vector D with coordinates]

Notice in this case I have replaced “north” with the y-axis and “east” with the x-axis. We now can express the direction of vector D as “28.3 degrees from the +x axis”.

Using the Cartesian system of x and y axes has a tremendous advantage. Although I have simply replaced “north” with “y” in this example, there is nothing in our use of x and y axes that requires “y” to be in the northern direction.

*When using x and y axes for directions, you always get to decide which direction is represented by each. For example, in some problems you will choose “y” to represent “up”; in others you might choose “y” to represent “down”. In other problems, the “y” direction might be at an angle to the vertical. Very important: you always get to choose x and y direction for each problem.*

This is what makes using the Cartesian system so important and versatile.

**A Vector Expressed as “Components”**

The most intuitive way to express a vector quantity is just as I have done above: its magnitude and angle. But vectors can also be expressed by their “components”. If we consider the example of vector D above, we can imagine we are trying to get from point A to point B. We can travel in a straight line
for a distance of 14.6 meters in a direction 28.3° north of east. This would be the shortest route. But when we express vectors we are only concerned with identifying the start and end point of the vector. If we know where point A is and we can find point B, we have all the information we need to draw our vector quantity.

If I start at point A and want to travel to point B, I can take a slightly longer route. I can first travel in the “east” direction, and at the appropriate point I can turn and travel in the “north” direction. We say that vector D has an “east component” and a “north component”. They look like this:

If we express these components using the Cartesian coordinate system, we get:

Since our vector is labeled “D” (this is just the name I chose to give it; we can use any name we like, as long as we give it a name), by convention we use the notation:

\[ x \text{ component of } D \text{ is } D_x \quad \text{and} \quad y \text{ component of } D \text{ is } D_y \]
We can calculate $D_x$ and $D_y$ using simple trigonometry. Note that the vector $D$ and its components form a right triangle. If we use “$D$” to represent the length of the hypotenuse, then we have:

$$\sin \theta = \frac{D_y}{D} \quad \text{and} \quad \cos \theta = \frac{D_x}{D}$$

By rearranging these expressions, we find:

$$D_y = D \sin \theta \quad \text{and} \quad D_x = D \cos \theta$$

We can use these last expressions to calculate the components, since we have values for $D$ and $\theta$.

$$D_y = D \sin \theta = (14.6 \text{ m}) \sin 28.3^\circ = 6.92 \text{ m}$$

$$D_x = D \cos \theta = (14.6 \text{ m}) \cos 28.3^\circ = 12.9 \text{ m}$$

We can now use this information to express our vector $D$ in terms of “components”:

$$D = 12.9 \text{ m \ in the x direction} + 6.92 \text{ m \ in the y direction}$$

This simply means that if you were to start at point A and follow these directions, you would arrive at point B. Notice that we started with a hypotenuse and angle and added the two sides to form a triangle. If we are only given components initially, we can add the hypotenuse to form a triangle.

**The magnitude of any vector is the hypotenuse of a right triangle!** The components of the vector are the sides of the triangle. You can always determine one expression from the other (i.e. magnitude and angle or “components”) by drawing the triangle!

When we express a vector as “components” we have to state the magnitude of the vector in each of the two perpendicular directions, which will usually be the x and y directions. Writing “in the x direction” and “in the y direction” can be a little tedious, so we use a bit of conventional shorthand:

“in the x direction” we abbreviate to “$\mathbf{i}$” and “in the y direction” we abbreviate to “$\mathbf{j}$”

Why do we use “$\mathbf{i}$” and “$\mathbf{j}$” for these? I have no idea, but just go with it. In my example above we would write:

$$D = 12.9 \text{ m \ in the x direction} + 6.92 \text{ m \ in the y direction}$$

as simply: $$D = (12.9 \text{ m}) \mathbf{i} + (6.92 \text{ m}) \mathbf{j}$$
We can now expand on this notation. A negative sign in front of any component indicates that component is in the negative direction of that axis. For example:

\[ E = (8.46 \text{ m}) \hat{i} - (9.32 \text{ m}) \hat{j} \]

means “8.46 m in the x direction and 9.32 m in the negative y direction”.

Note: when using the Cartesian system, angles are typically expressed in the range of \(0^\circ\) to \(360^\circ\) and are measured counterclockwise from the positive x axis.

Remember, to draw a vector you simply need to

- Identify the starting point
- Draw the components or magnitude and angle, whichever is given
- Identify the ending point
- Finish the triangle

**Adding Vectors**

We can now consider traveling from point A to point B and then onward to point C! Perhaps our first vector is from A to B and is given as in the example above. Our second vector tells us where to go when we leave point B and is given by:

\[ E = 10.2 \text{ meters at an angle of } 65.4 \text{ degrees north of east} \]

Following vector E brings us to point C, as in the diagram:

We can now “add” vectors D and E by considering their combined result: traveling from A to C. In compact notation, we can say that:

\[ F = D + E \quad \text{i.e. vector F is the sum of vector D and vector E} \]
This simply means that if you start at point A and first travel along vector D and then along vector E, you will arrive at the same point (i.e. point C) as if you had just traveled along vector F instead. Therefore, vector D plus vector E is equal to vector F. Consider the diagram below, where I have included vector F and changed to Cartesian notation:

How do we find vector F from the information given for D and E? The magnitude of every vector is the hypotenuse of a right triangle! This is the single most important thing to remember when working with vectors. We must draw triangles!

In the diagram below, I have included the components of vectors D, E and F. We have already calculated the components of vector D; we calculate the components of vector E in the same way.
Notice that there are three right triangles in the diagram, one for each of the three vectors. With the given information, you can:

- Calculate the x and y components of the given vectors, D and E
- Use the components of D and E to determine the components of F
- Use the components of F to find the magnitude and angle of F

The last step can be accomplished by using the Pythagorean theorem (to get the hypotenuse) and then using any trig function (because you know all three sides of the triangle) to determine the angles in the triangle. Using simple geometry, you can use the angles of the triangle to determine the angle vector F makes with the x axis.