Key

Name___________________________ Date________________

Midterm Score_________ Overall Grade__________

Math 35 Midterm 3
Fall 2012
Riverside City College

Instructions: All work is to be shown, done in pencil, legible, simplified and answers/solutions are to be boxed in the space provided. You are to work alone and any student caught cheating will receive a zero. You are allowed only a scientific calculator! Answers/solutions to word problems are to be written in a complete sentence with the correct units. Moreover, you will be allotted 1 hour and 30 minutes for this exam. If you finish early, then go back and check your work and answers. Students are not allowed to leave the room and return. So, if you have to use the restroom, please use it before starting the exam. Before you begin, remember to turn off all phones. Failure to comply with any of these instructions may result in a zero.
1. (4pts each) Graph the following function by making a "t-table" of at least 3-values and then state the Domain and Range in Interval Notation.

a) $f(x) = -\sqrt{2 - x} + 1$ 
Graph it here: 

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
</tbody>
</table>

Domain & Range: 
$D: (-\infty, 2]$  
$R: (-\infty, 1]$ 

b) $f(x) = 2\sqrt{x - 3} + 1$ 
Graph it here: 

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Domain & Range: 
$D: [-\infty, \infty)$  
$R: [-\infty, \infty)$
2. (3 pts each) Let \( r(x) = \frac{3x^3 - 2x + 7}{x - 3} \), find the quotient and remainder using the following method:

a) Long Division

\[
\begin{array}{c|cccc}
\text{x-3} & 3x^3 & 0x^2 & -2x & 7 \\
\hline
& 3x^3 & +9x^2 & -27x & +81 \\
\hline
& -3x^3 & +9x^2 & & \\
\hline
& 9x^2 & -2x & & \\
\hline
& -9x^2 & +27x & & \\
\hline
& 25x & +7 & & \\
\hline
& -25x & +75 & & \\
\hline
& 82 & & & \\
\end{array}
\]

\( q(x) = 3x^2 + 9x + 25 \)
\( r(x) = 82 \)

b) Synthetic Division

\[
\begin{array}{c|cccc}
3 & 3 & 0 & -2 & 7 \\
\hline
& 9 & & 27 & 75 \\
\hline
& 3 & 9 & 25 & 182 \\
\end{array}
\]

\( q(x) = 3x^2 + 9x + 25 \)
\( r(x) = 82 \)
3. (4pts each) Solve the following equations

a) \[ \frac{x-1}{2} + 1 = \frac{3}{x} \]

\[ x(x-1) + 1(2x) = 3(2) \]
\[ x^2 - x + 2x = 6 \]
\[ x^2 - x - 6 = 0 \]
\[ (x-3)(x+2) = 0 \]

\[ \therefore x = 3 \quad \text{or} \quad x = -2 \]

b) \[ \frac{17}{x-4} - \frac{10}{x+2} = 2 \]

\[ 17(x+2) - 10(x-4) = 2(x-4)(x+2) \]
\[ 17x + 34 - 10x + 40 = 2x^2 - 4x - 16 \]
\[ 7x + 74 = 2x^2 - 4x - 16 \]
\[ 0 = 2x^2 - 11x - 90 \]
\[ 0 = (2x + 9)(x - 10) \]

\[ \therefore x = -\frac{9}{2} \quad \text{or} \quad x = 10 \]
c) \( \frac{x-1}{x+3} - \frac{2x-1}{x-3} = \frac{-x+2}{x-3} \)

\[
\begin{align*}
(x-1)(x-3) - (2x-1)(x+3) &= (-x+2)(x+3) \\
x^2 - 4x + 3 - 2x^2 - 5x + 3 &= -x^2 - x + 6 \\
-x^2 - 9x + 6 &= -x^2 - x + 6 \\
-9x + 6 &= -x + 6 \\
0 &= 8x
\end{align*}
\]

\therefore x = 0


d) \( \frac{3}{x-2} + \frac{x-14}{2x^2-3x-2} - \frac{4}{2x+1} = 0 \)

\[
\begin{align*}
\frac{3}{x-2} + \frac{x-14}{(2x+1)(x-2)} - \frac{4}{(2x+1)} &= 0 \\
3(2x+1) + (x-14) - 4(x-2) &= 0 \\
6x + 3 + x - 14 - 4x + 8 &= 0 \\
3x - 3 &= 0 \\
3x &= 3 \\
\therefore x &= 1
\end{align*}
\]
e) \( \sqrt{2x-3} + 9 = x \)

\[
\sqrt{2x-3} = x - 9
\]

\[
2x - 3 = x^2 - 18x + 81
\]

\[
0 = x^2 - 20x + 84
\]

\[
(x - 14)(x - 6) = 0
\]

\[
\sqrt{26-3} + 9 = 6 \times
\]

\[
\text{Extraneous.}
\]

f) \( \sqrt{x^3 - 63} = x - 3 \)

\[
x^3 - 63 = (x - 3)^3
\]

\[
x^3 - 63 = (x - 3)(x^2 - 6x + 9)
\]

\[
x^3 - 63 = x^3 - 6x^2 + 9x - 3x^2 + 18x - 27
\]

\[
x^3 - 63 = x^3 - 9x^2 + 27x - 27
\]

\[
0 = -9x^2 + 27x + 36
\]

\[
0 = x^2 - 3x - 4
\]

\[
(x - 4)(x + 1) = 0
\]

\[
\sqrt{4 - 63} = 4 - 3 \checkmark
\]

\[
\sqrt{(-1)^3 - 63} = -1 - 3 \checkmark
\]

They both work!!!
g) \( \sqrt{x-7} + \sqrt{x} = 7 \)

\[
\sqrt{x-7} = 7 - \sqrt{x}
\]

\[
x - 7 = 49 - 14\sqrt{x} + x
\]

\[
-56 = -14\sqrt{x}
\]

\[
4 = \sqrt{x}
\]

\[
\boxed{x = 16}
\]

\[
\text{Check:} \quad \sqrt{16 - 7} + \sqrt{16} = 7 \quad \checkmark
\]

h) \( \sqrt{x+2} + \sqrt{2x} = \sqrt{18-x} \)

\[
(\sqrt{x+2} + \sqrt{2x})^2 = (\sqrt{18-x})^2
\]

\[
x + 2 + 2\sqrt{2x^2 + 4x} + 2x = 18 - x
\]

\[
2\sqrt{2x^2 + 4x} = 16 - 4x
\]

\[
\sqrt{2x^2 + 4x} = 8 - 2x
\]

\[
2x^2 + 4x = 64 - 32x + 4x^2
\]

\[
0 = 2x^2 - 36x + 64
\]

\[
0 = x^2 - 18x + 32
\]

\[
(x - 16)(x - 2) = 0
\]

\[
\therefore \boxed{x = 16 \text{ or } x = 2}
\]

\[
\text{Extraneous}
\]

\[
\sqrt{2+2} + \sqrt{2-2} = \sqrt{18-2} \quad \checkmark
\]
4. (5pts) It takes a man 3 hours to wash and wax the family car. If his teenage son helps him, it only takes 1 hour. How long would it take the son, working alone, to wash and wax the car? *Declare your variables and write your answers in a complete sentence with the correct units.*

\[ \frac{1}{3} + \frac{1}{x} = \frac{1}{1} \]

\[ x + 3 = 3x \]

\[ 2 = 2x \]

\[ x = \frac{3}{2} \text{ hrs} \]

The son can wash and wax the car in 1 hr and 30 mins.
5. (5pts) A kayaker can travel 1.2 miles downstream (with the current) in the same time it takes him to go 0.4 miles upstream (against the current). If the river current flows at 2 mph, what is the kayaker’s speed in still water? Declare your variables and write your answers in a complete sentence with the correct units.

\[ \text{let } r = \text{kayaker's speed in still water} \]

\[ \frac{1.2 \text{ miles}}{r+2} \to t_1 \]

\[ t_2 \leq \frac{0.4 \text{ miles}}{r-2} \]

Clearly, \( t_1 = t_2 \) (from the story!)

\[ \Rightarrow \frac{1.2}{r+2} = \frac{0.4}{r-2} \]

\[ 1.2(r-2) = 0.4(r+2) \]

\[ \frac{6}{5}(r-2) = \frac{2}{5}(r+2) \]

\[ \frac{6r}{5} - \frac{12}{5} = \frac{2r}{5} + \frac{4}{5} \]

\[ 6r - 12 = 2r + 4 \]

\[ 4r = 16 \]

\[ r = 4 \]

\[ \text{The kayaker's speed in still water is } 4 \text{ mph.} \]
6. (5pts) The resistance of a wire is directly proportional to the length of the wire and inversely proportional to the square of the diameter of the wire. If the resistance is 11.2 ohms in a 80-foot-long wire with diameter 0.01 inch, what is the resistance in a 160-foot-long wire with diameter 0.04 inch? Declare your variables and write your answers in a complete sentence with the correct units.

Let \( r = \text{resistance}, \ l = \text{length of wire}, \ d = \text{diameter} \)

\( r = \frac{k \cdot l}{d^2} \)

\( \Rightarrow 11.2 = \frac{k \cdot 80}{(0.01)^2} \)

\( k = \frac{(11.2) \cdot (0.01)^2}{80} \)

\( k = 0.000014 \)

\( r = \frac{0.000014 \cdot 160}{(0.04)^2} \)

\( r = 1.4 \)

\( \therefore\) The resistance is 1.4 ohms for a 160-ft long wire with 0.04 inch diameter.

Extra Credit: (2pts) Solve \( \sqrt[4]{x} = \frac{x}{\sqrt[4]{4}} \)

\( (\sqrt[4]{x})^4 = \left( \frac{x}{\sqrt[4]{4}} \right)^4 \)

\( x = \left( \frac{x}{\sqrt[4]{4}} \right)^2 \)

\( x = \frac{x^2}{16} \)

\( 16x = x^2 \)

\( 0 = x^2 - 16x \)

\( x(x - 16) = 0 \)

\( \therefore \) \( x = 0 \text{ or } x = 16 \)