Instructions: All work is to be shown, legible and answers/solutions are to be boxed in the space provided. You are to work alone and any student caught cheating will receive a zero. You will be allotted 2 hours and may only use a pencil and scientific calculator for this exam. Answers to word problems are to be written in a complete sentence with the correct units. Failure to comply with these instructions may result in a zero. Good luck and have some fun!
1. Let \( f(x) = \begin{cases} 
  x & \text{if } x < 1 \\
  3 & \text{if } x = 1 \\
  2 - x^2 & \text{if } 1 < x \leq 2 \\
  x - 3 & \text{if } x > 2 
\end{cases} \)

a) (3pts) Graph a detail graph of \( f \).

b) (1pt each) Evaluate the following limits, if it exist.

i) \( \lim_{x \to 1^-} f(x) \)   
ii) \( \lim_{x \to 1^+} f(x) \)   
iii) \( \lim_{x \to 1} f(x) \)   
iv) \( \lim_{x \to 2^-} f(x) \)   
v) \( \lim_{x \to 2^+} f(x) \)   
vi) \( \lim_{x \to 2} f(x) \)   


2. (2pts each) Evaluate the following limits, if it exists.

a) \( \lim_{x \to 0} \left( \frac{1 + x + \cos x}{3 \cos x} \right) \)

b) \( \lim_{x \to 3} \left( \frac{x^3 - 27}{x^2 - 9} \right) \)

c) \( \lim_{x \to 2} \left( \frac{4 - x^2}{3 - \sqrt{x^2 + 5}} \right) \)

d) \( \lim_{x \to 0} \left( \frac{1}{x \sqrt{1 + x}} - \frac{1}{x} \right) \)

e) \( \lim_{x \to 0} \left[ \frac{(2 + x)^{-1} - 2^{-1}}{x} \right] \)

f) \( \lim_{x \to \infty} \left( \frac{1 - e^x}{1 + 2e^x} \right) \)
3. Use the Squeeze Theorem to evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

4. Sketch a graph of a function $f$ for which $f(0) = f(2) = f(4) = 0$, $f'(1) = f'(3) = 0$, $f'(0) = f'(4) = 1$, $f'(2) = -1$, $\lim_{x \to \infty} f(x) = \infty$, and $\lim_{x \to -\infty} f(x) = -\infty$. 
5. Given the graph of $f$ below:

![Graph of f](image)

a) Sketch a graph of $f'$

b) Sketch a graph of $f''$
6. Let \( f(x) = x^2 \). Find a number \( \delta \) such that if \( |x - 2| < \delta \), then \( |x^2 - 4| < \frac{1}{4} \).

7. Use the precise definition of a limit (\( \varepsilon \delta \)-definition) to prove \( \lim_{x \to 2} x^2 = 4 \).
8. Show that \( f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases} \) is continuous on \((-\infty, \infty)\).

9. Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval
\[
\sin x = x^2 - x \text{ where } x \in (1, 2).
\]
10. Let \( f(x) = \frac{1}{4} x^2 - \pi \).

a) Use the definition of a derivative to find \( f'(x) \).

b) Find the equation of the tangent line (in slope-intercept form) at \( x = 2 \).