Instructions: All work is to be shown, legible, simplified and answers are to be boxed in the space provided. You are to work alone and any student caught cheating will receive a zero. You will be allotted 2 hours and may only use a pencil for this exam. Answers to word problems are to be written in a complete sentence with the correct units. Students are not allowed to leave and return. Thus, if you need to use the restroom, do so now!
1. (3pts) Use the $\varepsilon \delta$-definition to show $\lim_{(x,y) \to (0,0)} \frac{2x^3y}{x^2+y^2} = 0$. 
2. (3pts) Show that the given limit \( \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \) does not exist.
3. (3pts) Find the critical points of \( f(x,y) = x^3 + 3xy^2 + 3y^2 - 15x + 2 \) and determine their nature.
4. (2 pts each) Suppose \( w = g(x,y,z) \) is differentiable. Let \( x = r \cos \theta \), \( y = r \sin \theta \), and \( z = t \). Calculate the following partial derivatives.

a) \( \frac{\partial w}{\partial r} \)

b) \( \frac{\partial w}{\partial \theta} \)

c) \( \frac{\partial w}{\partial t} \)
5. (4pts) One of the most important partial-differential equations of mathematical physics is Laplace’s equation in \( \mathbb{R}^2 \), given by \( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \) or \( f_{xx} + f_{yy} = 0 \). Verify that \( f(x, y) = \ln(x^2 + y^2) \) satisfies Laplace’s equation.
6. (4pts) Find the equation of the tangent plane and symmetric equations of the normal line to the surface $xy^2 - yz^2 + zx^2 = 1$ at $(1, 1, 1)$. 
7. (2pts each) Find the equation of the line tangent (in symmetric form) to the surface \( z = x^3 - 4y^3 \) at the point \((1, -1, 5)\) that lies in the plane

(a) \( x = 1 \)

(b) \( y = -1 \)
8. (4pts) The height of a mountain is given by \( h(x,y) = 3000 - 2x^2 - y^2 \), where the \( y \)-axis points east, the \( x \)-axis points north, and all distances are measured in meters. Suppose Mr. Chiek is at the point \((30, -20, 800)\).

a) (3pts) If Mr. Chiek moves in the southwest directions, will he ascend or descend?

b) (1pt) In what direction should Mr. Chiek move so he is to ascend most rapidly?
9. (4pts) Use the method of the Lagrange Multiplier to find the maximum and minimum values of $x + y + z$ if $(x, y, z)$ lies on the sphere $x^2 + y^2 + z^2 = 1$. 