Math 1C Midterm 3
Fall 2015
Riverside City College
Use as a Review!!!

Alcohol and calculus don’t mix.
Never drink and derive.

Please read and follow the following directions:
1. Do all work on the space provide.
2. Box your answer when possible.
3. All answers are to be exact and simplified.
4. No Electronic Devices!
5. Students are not allowed to leave and return when the exam starts.
   Thus, if you need to use the restroom, do so now!
6. If time permits, go back and check your work!
7. Cheating is prohibited, so please keep your eyes on your paper!
1. (4pts) Find the volume of the region bounded above by the surface $z = 2\sin x \cos y$ and below by the rectangle $\mathcal{R} = \{(x,y) : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{4}\}$.
2. (4pts) Evaluate \( \int_{1}^{\infty} \int_{-1}^{1} \frac{1}{xy} dy dx \).
3. (4pts) The double integral \( \int_0^\pi \int_0^{\cos x} dy \, dx \) gives the area of a region in the xy-plane. Sketch the region, label each bounding curve with its equation, and give the coordinates of the points where the curves intersect. Then find the area of the region.
4. (4pts) Change the Cartesian integral $\int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{0} \left( \frac{2}{1 + \sqrt{x^2 + y^2}} \right) dy \, dx$ into an equivalent polar integral. Then evaluate the polar integral.
5. (4pts) Below is the region of integration of the integral

\[ \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz \, dy \, dx. \]

Rewrite the integral as an equivalent iterated integral in the order

a) \( dy \, dz \, dx \)

b) \( dy \, dx \, dz \)

c) \( dx \, dy \, dz \)

d) \( dx \, dz \, dy \)

e) \( dz \, dx \, dy \)
6. (4pts) Find the region in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$. 
7. (4pts) Let $D$ be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$.

a) Sketch the solid

b) Set up the triple integral in cylindrical coordinates that give the volume of $D$ using the following orders of integration.

i) $dzdrd\theta$

ii) $drdzd\theta$

iii) $d\theta dzdr$
8. (4pts) Find the volume of the portion of the solid sphere $\rho \leq 5$ that lies between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{2\pi}{3}$. 
Extra Credit: (2pts) Mr. Chiek was bored one day and decided to put a hole in a sphere. That is, a circular cylindrical hole is bored through a solid sphere, the axis of the hole being a diameter of the sphere. The volume of the remaining solid is

$$V = 2\int_0^{2\pi} \int_0^\pi \int_0^{\sqrt{4-z^2}} r \, dr \, dz \, d\theta.$$  

How much was bored out of the sphere?