Chapter 8 - Hypothesis Testing

Note: Graphs are not to scale and are intended to convey a general idea. Answers may vary due to rounding.

EXERCISE SET 8-1

1. The null hypothesis is a statistical hypothesis that states there is no difference between a parameter and a specific value or there is no difference between two parameters. The alternative hypothesis specifies a specific difference between a parameter and a specific value, or that there is a difference between two parameters. Examples will vary.

2. A type I error occurs by rejecting the null hypothesis when it is true. A type II error occurs when the null hypothesis is not rejected and it is false. They are related in that decreasing the probability of one type of error increases the probability of the other type of error.

3. A statistical test uses the data obtained from a sample to make a decision as to whether or not the null hypothesis should be rejected.

4. A one-tailed test indicates the null hypothesis should be rejected when the test statistic value is in the critical region on one side of the mean. A two-tailed test indicates the null hypothesis should be rejected when the test statistic value is in either critical region on both sides of the mean.

5. The critical region is the region of values of the test-statistic that indicates a significant difference and the null hypothesis should be rejected. The non-critical region is the region of values of the test-statistic that indicates the difference was probably due to chance, and the null hypothesis should not be rejected.


7. Type I is represented by α, type II is represented by β.

8. When the difference between the sample mean and the hypothesized mean is large, then the difference is said to be significant and probably not due to chance.

9. A one-tailed test should be used when a specific direction, such as greater than or less than, is being hypothesized, whereas when no direction is specified, a two-tailed test should be used.

10. Hypotheses can only be proved true when the entire population is used to compute the test statistic. In most cases, this is impossible.

11. a. ± 1.96

b. −2.33

c. +2.58
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11. continued
d.  +2.33

e.  −1.65

12. continued
d.  −1.75

e.  +2.05

EXERCISE SET 8-2

1.  
   \( H_0: \mu = 305 \)

13.
a.  \( H_0: \mu = 24.6 \)
    \( H_1: \mu \neq 24.6 \)

b.  \( H_0: \mu = \$51,497 \)
    \( H_1: \mu \neq \$51,497 \)

c.  \( H_0: \mu = 25.4 \)
    \( H_1: \mu > 25.4 \)

d.  \( H_0: \mu = 88 \)
    \( H_1: \mu < 88 \)

e.  \( H_0: \mu = 8.2 \)
    \( H_1: \mu \neq 8.2 \)
1. continued

Hₜ:  μ > 305  (claim)

C. V.  = 1.65  σ = \sqrt{3.6} = 1.897

\[ z = \frac{\bar{x} - \mu}{\sigma} = \frac{306.2 - 305}{1.897} = 4.69 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average depth has increased. Many factors could contribute to the increase, including warmer temperatures and higher than usual rainfall.

2. 

H₀:  μ = 130  
H₁:  μ > 130  (claim)

C. V.  = 1.65

\[ z = \frac{\bar{x} - \mu}{\sigma} = \frac{162 - 130}{38.2} = 5.92 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average number of friends is more than 130.

3. continued

H₀:  μ = $24 billion  
H₁:  μ > $24 billion  (claim)

C. V.  = +1.65  \bar{x} = $31.5  s = $28.7

\[ z = \frac{\bar{x} - \mu}{s} = \frac{31.5 - 24}{3.2} = 2.17 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average revenue exceeds $24 billion.

4. 

H₀:  μ = 8.5  
H₁:  μ ≠ 8.5  (claim)

C. V.  = ±1.96

\[ z = \frac{\bar{x} - \mu}{s} = \frac{9.6 - 8.5}{3.2} = 2.17 \]

Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the average number of movies.

5. 

H₀:  μ = 30.9  
H₁:  μ ≠ 30.9  (claim)

C. V.  = ±2.58

\[ z = \frac{\bar{x} - \mu}{s} = \frac{12.1 - 30.9}{3.6} = 1.89 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that average number of hours differs from 30.9.
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6.  
\(H_0: \mu = \$117.91\)  
\(H_1: \mu \neq \$117.91\) (claim)  
\[C. V. = \pm 1.65\]  
\[z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\$122.57 - \$117.91}{\$6/\sqrt{30}} = 1.40\]  
\[\begin{array}{c}
-1.65 \\
0 \\
1.65 \\
1.40
\end{array}\]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the cost per square foot differs from \$117.91.

7.  
\(H_0: \mu = 29\)  
\(H_1: \mu \neq 29\) (claim)  
\[C. V. = \pm 1.96\]  
\[z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{29.45 - 29}{2.61/\sqrt{30}} = 0.944\]  
\[\begin{array}{c}
1.96 \\
0 \\
1.96 \\
0.944
\end{array}\]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average height differs from 29 inches.

8.  
\(H_0: \mu = \$59,593\)  
\(H_1: \mu < \$59,593\) (claim)  
\[C. V. = -2.33\]  
\[z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\$58.800 - \$59.593}{\$1500/30} = -2.90\]  
\[\begin{array}{c}
-1.96 \\
0 \\
1.96 \\
0.62
\end{array}\]

Do not reject the null hypothesis. There is not enough evidence to support the claim.

8. continued

\[\begin{array}{c}
\uparrow -2.33 \\
0 \\
-2.90
\end{array}\]

Reject the null hypothesis. There is enough evidence to support the claim that the average income is less than \$59,593.

9.  
\(H_0: \mu = \$8121\)  
\(H_1: \mu > \$8121\) (claim)  
\[C. V. = 2.33\]  
\[z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\$8350 - \$8121}{\$1500/30} = 1.93\]  
\[\begin{array}{c}
0 \\
\uparrow 2.33 \\
1.93
\end{array}\]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average operating cost has increased.

10.  
\(H_0: \mu = 6.698\)  
\(H_1: \mu \neq 6.698\) (claim)  
\[s = 5.5\text{ inches} = 0.458\text{ feet}\]  
\[C. V. = \pm 1.96\]  
\[z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{6.75 - 6.698}{0.458/\sqrt{30}} = 0.62\]  
\[\begin{array}{c}
-1.96 \\
0 \\
1.96 \\
0.62
\end{array}\]

Do not reject the null hypothesis. There is not enough evidence to support the claim.
10. continued that the average height differs from 6.698 feet.

11. 
   \[ H_0: \mu = 150 \]
   \[ H_1: \mu > 150 \] (claim)

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{152.59 - 150}{2.33 / \sqrt{38}} = 1.48 \]

   Do not reject the null hypothesis. There is not enough evidence to support the claim that the average cost is greater than $150.

12. 
   \[ H_0: \mu = 10,337 \]
   \[ H_1: \mu \neq 10,337 \] (claim)

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{10,798 - 10,337}{3.5 / \sqrt{40}} \]

   Since 3.62 is outside the critical values of \( \alpha = 0.10, 0.05, \) and \( 0.01 \), reject the null hypothesis. There is a significant difference in student expenditures.

13. 
   \[ H_0: \mu = 60.35 \]
   \[ H_1: \mu < 60.35 \] (claim)

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{55.4 - 60.35}{3.5 / \sqrt{40}} = -4.82 \]

   Reject the null hypothesis. There is enough evidence to support the claim that the mean is 52. The researcher's claim is not valid.

14. 
   \[ H_0: \mu = 34.9 \]
   \[ H_1: \mu \neq 34.9 \] (claim)

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{28.5 - 34.9}{1.96 / \sqrt{40}} = -4.55 \]

   Reject the null hypothesis. There is enough evidence to support the claim that the average stay differs from 34.9 months.

15. 
   a. Do not reject.
   b. Reject.
   c. Do not reject.
   d. Reject.
   e. Reject.

16. 
   \[ H_0: \mu = 52 \] (claim)
   \[ H_1: \mu \neq 52 \]

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{56.3 - 52}{1.96 / \sqrt{40}} = 8.69 \]

   The area corresponding to \( z = 8.69 \) is \( 0.9999 \). Then P-value < 0.01. Hence, the null hypothesis should be rejected. There is enough evidence to reject the claim that the mean is 52. The researcher's claim is not valid.

17. 
   \[ H_0: \mu = 264 \]
   \[ H_1: \mu < 264 \] (claim)

   C. V. \[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{262.3 - 264}{2.53 / \sqrt{40}} = -2.53 \]

   The area corresponding to \( z = 2.53 \) is 0.9943. The P-value is \( 1 - 0.9943 = 0.0057 \). The decision is to reject the null hypothesis since 0.0057 < 0.01. There is
17. continued
enough evidence to support the claim that
the average stopping distance is less than
264 feet. (TI: P-value = 0.0056)

18.
\[ H_0: \mu = 40 \]
\[ H_1: \mu < 40 \quad \text{(claim)} \]
\[ \bar{X} = 29.26 \quad \sigma = 30.9 \]
\[ z = \frac{\bar{X} - \mu}{\sigma} = \frac{29.26 - 40}{30.9} = -2.458 \]

The area corresponding to \( z = -2.458 \) is
0.0069. The decision is to reject the null hypothesis since 0.0069 < 0.01. There is
enough evidence to support the claim that the average number of copies is less than 40.
(TI: P-value = 0.00699)

19.
\[ H_0: \mu = 546 \]
\[ H_1: \mu < 546 \quad \text{(claim)} \]
\[ z = \frac{\bar{X} - \mu}{\sigma} = \frac{544.8 - 546}{36} = -2.40 \]

The area corresponding to \( z = -2.40 \) is
0.0082. Thus, P-value = 0.0082. The
decision is to reject the null hypothesis since 0.0082 < 0.01. There is
enough evidence to support the claim that the number of calories burned is less than 546.
(TI: P-value = 0.0082)

20.
\[ H_0: \mu = 800 \quad \text{(claim)} \]
\[ H_1: \mu \neq 800 \]
\[ z = \frac{\bar{X} - \mu}{\sigma} = \frac{793 - 800}{\sqrt{12}} = -2.61 \]

The area corresponding to \( z = -2.61 \) is
0.0045. The P-value is found by multiplying by 2 since this is a two-tailed test. Hence,
\( 2(0.0045) = 0.0090 \). The decision is to reject the null hypothesis since 0.009 < 0.01. There is enough evidence to reject the null hypothesis that the breaking strength is 800 pounds.

21. continued
The decision is do not reject the null hypothesis since P-value > 0.05. There is
not enough evidence to support the claim that the mean differs from 444.
(TI: P-value = 0.0886)

22.
\[ H_0: \mu = 65 \quad \text{(claim)} \]
\[ H_1: \mu \neq 65 \]
\[ z = \frac{\bar{X} - \mu}{\sigma} = \frac{544.8 - 65}{\sqrt{36}} = -1.21 \]

The area corresponding to \( z = -1.21 \) is
0.1131. The P-value is \( 2(0.1131) = 0.2262 \). The
decision is do not reject the null hypothesis since 0.2262 > 0.10. Hence, there is not enough evidence to reject the claim that the average is 65 acres.

23.
\[ H_0: \mu = 30,000 \quad \text{(claim)} \]
\[ H_1: \mu \neq 30,000 \]
\[ z = \frac{\bar{X} - \mu}{\sigma} = \frac{544.8 - 30,000}{\sqrt{1484}} = 1.71 \]

The area corresponding to \( z = 1.71 \) is
0.9564. The P-value is \( 2(1 - 0.9564) = 2(0.0436) = 0.0872 \). The decision is to reject the null hypothesis at \( \alpha = 0.10 \) since 0.0872 < 0.10. The conclusion is that there is enough evidence to reject the claim that customers are adhering to the recommendation. Yes, the 0.10 significance level is appropriate. (TI: P-value = 0.0868)

24.
\[ H_0: \mu = 60 \quad \text{(claim)} \]
\[ H_1: \mu \neq 60 \]
\[ \bar{X} = 59.93 \quad s = 13.42 \]
\[ z = \frac{\bar{X} - \mu}{s} = \frac{59.93 - 60}{13.42} = -0.03 \]

The P-value is \( 2(0.4880) = 0.9760 \). (TI: P-value = 0.9763). Since 0.9760 > 0.05, the
decision is do not reject the null hypothesis. There is not enough evidence to reject the claim that the average number of speeding tickets is 60.

25. continued
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25. continued
\[ X = 5.025 \quad s = 3.63 \]
\[ z = \frac{\bar{x} - \mu}{s} = \frac{5.025 - 10}{3.63} = -8.67 \]

The area corresponding to \(-8.67\) is less than 0.0001. Since 0.0001 < 0.05, the decision is to reject the null hypothesis. There is enough evidence to support the claim that the average number of days missed per year is less than 10.

26.
Reject the claim at \(\alpha = 0.05\) but not at \(\alpha = 0.01\). There is no contradiction since the value of \(\alpha\) should be chosen before the test is conducted.

27.
The mean and standard deviation are found as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>f</th>
<th>Xf</th>
<th>Xf^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.35 - 8.43</td>
<td>2</td>
<td>8.39</td>
<td>16.78</td>
</tr>
<tr>
<td>8.44 - 8.52</td>
<td>6</td>
<td>8.48</td>
<td>50.88</td>
</tr>
<tr>
<td>8.53 - 8.61</td>
<td>12</td>
<td>8.57</td>
<td>102.84</td>
</tr>
<tr>
<td>8.62 - 8.70</td>
<td>18</td>
<td>8.66</td>
<td>155.88</td>
</tr>
<tr>
<td>8.71 - 8.79</td>
<td>10</td>
<td>8.75</td>
<td>87.50</td>
</tr>
<tr>
<td>8.80 - 8.88</td>
<td>2</td>
<td>8.84</td>
<td>17.68</td>
</tr>
</tbody>
</table>

\[ X = \frac{\sum Xf}{n} = \frac{531.56}{50} = 8.63 \]
\[ s = \sqrt{\frac{\sum Xf^2 - \left(\frac{\sum Xf}{n}\right)^2}{n-1}} = \sqrt{3725.4224 - \left(\frac{531.56}{50}\right)^2} \]
\[ s = 0.105 \]

\[ H_0: \mu = 8.65 \quad \text{claim} \]
\[ H_1: \mu \neq 8.65 \]

\[ 0.9208 \]

\[ z = \frac{\bar{x} - \mu}{s} = \frac{8.63 - 8.65}{0.105} = -1.35 \]

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the average hourly wage of the employees is $8.65.

EXERCISE SET 8-3

1. continued
median, and mode are all equal to 0 and they are located at the center of the distribution. The \(t\) distribution differs from the standard normal distribution in that it is a family of curves, the variance is greater than 1, and as the degrees of freedom increase the \(t\) distribution approaches the standard normal distribution.

2. The degrees of freedom are the number of values that are free to vary after a sample statistic has been computed. They tell the researcher which specific curve to use when a distribution consists of a family of curves.

3. a. d. f. = 9 \quad C. V. = + 1.833
b. d. f. = 17 \quad C. V. = + 1.740
c. d. f. = 5 \quad C. V. = - 3.365
d. d. f. = 8 \quad C. V. = + 2.306

4. a. d. f. = 14 \quad C. V. = \pm 1.761
b. d. f. = 22 \quad C. V. = - 2.819
c. d. f. = 27 \quad C. V. = \pm 2.771
d. d. f. = 16 \quad C. V. = \pm 2.583

5. a. 0.01 < P-value < 0.025 (0.018)
b. 0.05 < P-value < 0.10 (0.062)
c. 0.10 < P-value < 0.25 (0.123)
d. 0.10 < P-value < 0.20 (0.138)

6. a. P-value < 0.005 (0.003)
b. 0.10 < P-value < 0.25 (0.158)
c. P-value = 0.05 (0.05)
d. P-value > 0.25 (0.261)

7. \[ H_0: \mu = 200 \quad \text{claim} \]
\[ H_1: \mu < 200 \]
\[ C. V. = -1.833 \quad \text{d. f.} = 9 \]
\[ t = \frac{\bar{x} - \mu}{s} = \frac{185.2 - 200}{\sqrt{\frac{9}{8}}} = -4.680 \]
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7. continued

$\uparrow - 1.833 \quad 0$
$- 4.680$

Reject the null hypothesis. There is enough evidence to support the claim that the average number of seeds is less than 200.

8.
$H_0: \mu = 5400$
$H_1: \mu < 5400$ (claim)

C. V. $= - 2.052 \quad d. f. = 27$
$t = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{5250 - 5400}{\sqrt{28}} = - 1.262$

$\downarrow - 2.052 \quad 0$
$- 1.262$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average cost is less than $5400.

9. continued

Reject the null hypothesis. There is enough evidence to reject the claim that the average height of the buildings is at least 700 feet.

10.
$H_0: \mu = 50,000$
$H_1: \mu > 50,000$ (claim)

C. V. $= 1.440 \quad d. f. = 6$
$\bar{x} = 50,363.57 \quad s = 1113.16$
$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{50,363.57 - 50,000}{1113.16} = 0.864$

$0 \quad \uparrow 1.440$
$0.864$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average number of words is greater than 50,000.

11.
$H_0: \mu = 73$
$H_1: \mu > 73$ (claim)

C. V. $= 2.821 \quad d. f. = 9$
$\bar{x} = 123.5 \quad s = 39.303$
$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{123.5 - 73}{39.303} = 4.063$

$0 \quad 2.821 \quad \uparrow$
$4.063$

Reject the null hypothesis. There is enough evidence to support the claim that the average viewing time is longer than 73 minutes.

12.
$H_0: \mu = 110$
$H_1: \mu > 110$ (claim)

C. V. $= 2.624 \quad d. f. = 14$
12. continued

\[ \bar{X} = 137.333 \quad s = 24.118 \]
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{137.333 - 110}{24.118/\sqrt{15}} \approx 4.389 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average number of calories is greater than 110.

13.

\( H_0: \mu = 54.8 \)
\( H_1: \mu > 54.8 \) (claim)

C. V. = 1.761 d. f. = 14
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{56.11 - 54.8}{6.97/\sqrt{10}} \approx 3.058 \]

Reject the null hypothesis. There is enough evidence to support the claim that the cost to produce an action movie is more than $54.8 million.

14.

\( H_0: \mu = 36 \)
\( H_1: \mu \neq 36 \) (claim)

C. V. = ± 2.807 d. f. = 23
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{42.136 - 36}{10.85/\sqrt{23}} \approx 5.638 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average stipend differs from $15,000.

15.

\( H_0: \mu = 50.07 \)
\( H_1: \mu > 50.07 \) (claim)

C. V. = 1.833 d. f. = 9
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{56.11 - 50.07}{6.97/\sqrt{10}} \approx 2.741 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average bill has increased.

16.

\( H_0: \mu = 15,000 \)
\( H_1: \mu \neq 15,000 \) (claim)

C. V. = ± 2.201 d. f. = 11
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{14,347.17 - 15,000}{2048.54/\sqrt{12}} \approx -1.104 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average usage differs from 36.

17.

\( H_0: \mu = 7.89 \)
\( H_1: \mu > 7.89 \) (claim)

C. V. = 2.624 d. f. = 14
17. continued
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{811.09 - 57.89}{\sqrt{8}} = 2.550 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average cost is greater than $7.89.

18.
H₀: \( \mu = 2.27 \)
H₁: \( \mu \neq 2.27 \) (claim)

C. V. = ± 2.093 d. f. = 19
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2.98 - 2.27}{\sqrt{19}} = 3.240 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average call differs from 2.27 minutes.

19. continued

H₀: \( \mu = 25.4 \)
H₁: \( \mu < 25.4 \) (claim)

C. V. = -1.318 d. f. = 24
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{22.1 - 25.4}{\sqrt{24}} = -3.11 \]

Since P-value < 0.01, reject the null hypothesis. There is enough evidence to support the claim that the mean number of jobs is not 9.2. One reason why a person may not give the exact number of jobs is that he

19. continued

Reject the null hypothesis. There is enough evidence to support the claim that the commute time is less than 25.4 minutes.

H₀: \( \mu = 3.18 \)
H₁: \( \mu \neq 3.18 \) (claim)

\[ \bar{x} = 3.833 \quad s = 1.4346 \]
d. f. = 24 C. V. = ± 2.069
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{3.833 - 3.18}{\sqrt{24}} = 2.2299 \text{ or } 2.23 \]

(TI answer 2.231)

Since P-value > 0.05, do not reject the null hypothesis. There is not enough evidence to support the claim that the mean is not 5.8.

20. 
H₀: \( \mu = 5.8 \)
H₁: \( \mu \neq 5.8 \) (claim)

\[ \bar{x} = 3.833 \quad s = 1.4346 \]
d. f. = 23 C. V. = ± 2.093
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{3.833 - 5.8}{\sqrt{23}} = -3.462 \]

Since P-value < 0.01, reject the null hypothesis. There is enough evidence to support the claim that the mean is 9.2.

21. 
H₀: \( \mu = 9.2 \) (claim)
H₁: \( \mu \neq 9.2 \)

\[ \bar{x} = 8.25 \quad s = 5.06 \]
d. f. = 7 P-value > 0.50 (0.6121)
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{8.25 - 9.2}{\sqrt{7}} = -0.531 \]

Since P-value > 0.50, do not reject the null hypothesis. There is not enough evidence to support the claim that the mean number of jobs is 9.2. One reason why a person may not give the exact number of jobs is that he
22. continued
or she may have forgotten about a particular job.

23.  
\[ H_0: \mu = 123 \]
\[ H_1: \mu \neq 123 \quad \text{(claim)} \]
d. f. = 15  
P-value < 0.01  
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{119 - 123}{5.3/\sqrt{15}} = -3.019 \]

Since P-value < 0.05, reject the null hypothesis. There is enough evidence to support the claim that the mean is not 123 gallons.

EXERCISE SET 8-4

1.  
Answers will vary.

2.  
The proportion of A items can be considered a success whereas the proportion of items that are not included in A can be considered a failure.

3.  
np ≥ 5 and nq ≥ 5

4.  
\[ \mu = p \quad \sigma = \sqrt{pq/n} \]

5.  
\[ H_0: p = 0.456 \]
\[ H_1: p \neq 0.456 \quad \text{(claim)} \]
\[ \hat{p} = \frac{60}{110} = 0.545 \quad p = 0.456 \quad q = 0.544 \]
C. V. = ± 1.65
\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.545 - 0.456}{\sqrt{\frac{0.456(1-0.456)}{110}}} = 1.87 \]
(TI: \( z = 1.884 \))

0  2.05  ↑ 2.357

Reject the null hypothesis. There is enough evidence to support the claim that the proportion of baseball fans is greater than 36%.

8.  
\[ H_0: p = 0.279 \]
\[ H_1: p > 0.279 \quad \text{(claim)} \]
\[ \hat{p} = \frac{45}{120} = 0.375 \quad p = 0.279 \quad q = 0.721 \]
C. V. = 1.65
\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.375 - 0.279}{\sqrt{\frac{0.279(1-0.279)}{120}}} = 2.35 \]
8. continued

\begin{align*}
0 & \quad 1.65 \quad \uparrow \quad 2.35 \\
\text{Reject the null hypothesis. There is enough} \\
\text{evidence to conclude that the proportion of} \\
\text{female physicians is higher than 27.9%}.
\end{align*}

9.

\begin{align*}
H_0: & \quad p = 0.30 \\
H_1: & \quad p \neq 0.30 \quad \text{(claim)} \\
\hat{p} = & \quad \frac{45}{130} = 0.346 \quad p = 0.30 \quad q = 0.70 \\
C. V. & = \pm 1.96 \\
z & = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.346 - 0.3}{\sqrt{\frac{0.3(0.7)}{130}}} = 1.145 \\
\text{(TI: } z = 1.148) \\
\end{align*}

\begin{align*}
-1.96 & \quad 0 \quad \uparrow \quad 1.96 \\
\text{Do not reject the null hypothesis. There is} \\
\text{not enough evidence to support the claim} \\
\text{that the proportion of open or unlocked} \\
\text{window or door burglaries differs from 30%}.
\end{align*}

10. continued

\begin{align*}
\text{Do not reject the null hypothesis. There is} \\
\text{not enough evidence to support the claim} \\
\text{that the percentage differs from the national rate.}
\end{align*}

11.

\begin{align*}
H_0: & \quad p = 0.32 \\
H_1: & \quad p \neq 0.32 \quad \text{(claim)} \\
\hat{p} = & \quad \frac{420}{423} = 0.997 \quad p = 0.32 \quad q = 0.68 \\
C. V. & = \pm 2.58 \\
z & = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.997 - 0.32}{\sqrt{\frac{0.32(0.68)}{423}}} = -2.68 \\
\text{(TI: } z = -2.62) \\
\end{align*}

\begin{align*}
-2.58 & \quad 0 \quad 2.58 \quad \uparrow \quad 3.61 \\
\text{Do not reject the null hypothesis. There is} \\
\text{not enough evidence to support the claim} \\
\text{that the percentage is less than 83%}.
\end{align*}

12.

\begin{align*}
H_0: & \quad p = 0.83 \\
H_1: & \quad p < 0.83 \quad \text{(claim)} \\
\hat{p} = & \quad \frac{240}{300} = 0.8 \quad p = 0.83 \quad q = 0.17 \\
C. V. & = -1.65 \\
z & = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.8 - 0.83}{\sqrt{\frac{0.83(0.17)}{300}}} = -1.38 \\
\text{(TI: } z = -1.38) \\
\end{align*}

\begin{align*}
-1.65 & \quad \uparrow \quad 0 \\
\text{Do not reject the null hypothesis. There is} \\
\text{not enough evidence to support the claim} \\
\text{that the percentage is less than 83%}.
\end{align*}

13.

\begin{align*}
H_0: & \quad p = 0.54 \quad \text{(claim)} \\
H_1: & \quad p \neq 0.54 \\
\hat{p} = & \quad \frac{36}{60} = 0.6 \quad p = 0.54 \quad q = 0.46 \\
\end{align*}
13. continued

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.6-0.54}{\sqrt{\frac{0.54(0.46)}{60}}} = 0.93 \]

Area = 0.8238

P-value = 2(1 - 0.8238) = 0.3524

Since P-value > 0.01, do not reject the null hypothesis. There is enough evidence to support the claim that 54% of kids had a snack after school. Yes, a healthy snack should be made available for children to eat after school. (TI: P-value = 0.3511)

14.

\( \hat{p} = \frac{115}{200} = 0.575 \)

\( p = 0.517 \)

\( q = 0.483 \)

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.575-0.517}{\sqrt{\frac{0.517(0.483)}{200}}} = 1.64 \]

Area = 0.9495

P-value = 2(1 - 0.9495) = 0.101

Since P-value > 0.05, do not reject the null hypothesis. There is not enough evidence to reject the claim that 51.7% of homes in America were heated by natural gas. The evidence supports the claim. The conclusion could be different if the sample is taken in an area where natural gas is not commonly used to heat homes.

15.

\( \hat{p} = \frac{50}{50} = 0.1667 \)

\( p = 0.18 \)

\( q = 0.82 \)

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.1667-0.18}{\sqrt{\frac{0.18(0.82)}{50}}} = -0.60 \]

P-value = 0.2743 (TI: P-value = 0.2739)

Since P-value > 0.05, do not reject the null hypothesis. There is not enough evidence to reject the claim that 18% of all high school students smoke at least a pack of cigarettes a day.

16. continued

Since P-value > 0.10, do not reject the null hypothesis. There is not enough evidence to reject the claim that 14% of men use exercise to relieve stress. The results cannot be generalized to all adults since only men were surveyed.

17.

\( H_0: p = 0.67 \)

\( H_1: p \neq 0.67 \) (claim)

\( \hat{p} = \frac{82}{100} = 0.82 \)

\( p = 0.67 \)

\( q = 0.33 \)

C. V. = \( \pm 1.96 \)

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.82-0.67}{\sqrt{\frac{0.67(0.33)}{100}}} = 3.19 \]

Reject the null hypothesis. There is enough evidence to support the claim that the percentage is not 67%.

18.

\( H_0: p = 0.6 \)

\( H_1: p < 0.6 \) (claim)

\( \hat{p} = \frac{26}{50} = 0.52 \)

\( p = 0.6 \)

\( q = 0.4 \)

C. V. = \(-1.65 \)

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.52-0.6}{\sqrt{\frac{0.6(0.4)}{50}}} = -1.15 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the percentage of paid assistantships is less than 60%.

19.

\( H_0: p = 0.576 \)

\( H_1: p < 0.576 \) (claim)
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19. continued
\[ \hat{p} = \frac{17}{36} = 0.472 \quad p = 0.576 \quad q = 0.424 \]
C. V. = \[ \frac{\hat{p}(1-\hat{p})}{n} = \frac{0.472(1-0.472)}{36} = -1.65 \]
\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = -1.26 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the percentage of injuries during practice is less than 57.6%.

20.
H_0: p = 0.7
H_1: p > 0.7 (claim)
\[ \hat{p} = \frac{204}{250} = 0.816 \quad p = 0.7 \quad q = 0.3 \]
C. V. = 2.33
\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.816 - 0.7}{\sqrt{\frac{0.7(0.3)}{250}}} = 4.00 \]

Reject the null hypothesis. There is enough evidence to support the claim that the percentage of college students who recycle is greater than 70%.

21.
This represents a binomial distribution with p = 0.50 and n = 9. The P-value is
\[ 2 \cdot P(X \leq 3) = 2(0.254) = 0.508 \]
Since P-value > 0.10, the conclusion that the coin is not balanced is probably false. The answer is no.

22. continued
This represents a binomial distribution with p = 0.20 and n = 15. The P-value is
\[ 2 \cdot P(X > 5) = 2(0.061) = 0.122, \text{ which is} \]
greater than \( \alpha = 0.10 \). Do not reject the null hypothesis. Here is not enough evidence to conclude that the proportions have changed.

23.
\[ z = \frac{X - \mu}{\sigma} \]
\[ z = \frac{X - np}{\sqrt{npq}} \]
\[ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \]

EXERCISE SET 8-5

1.
a. H_0: \sigma^2 = 225
H_1: \sigma^2 > 225
C. V. = 27.587 d. f. = 17

\[ \begin{array}{c}
\text{0} \\
27.587
\end{array} \]

b. H_0: \sigma^2 = 225
H_1: \sigma^2 < 225
C. V. = 14.042 d. f. = 22

\[ \begin{array}{c}
\text{0} \\
14.042
\end{array} \]

c. H_0: \sigma^2 = 225
H_1: \sigma^2 \neq 225
C. V. = 5.629, 26.119 d. f. = 14
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1c. continued

\[
\begin{array}{c}
0 & 5.629 & 26.119 \\
\end{array}
\]

d. \( H_0: \sigma^2 = 225 \)
\( H_1: \sigma^2 \neq 225 \)

C. V. = 2.167, 14.067 \hspace{1cm} d. f. = 7

2c. continued

\[
\begin{array}{c}
0 & 3.074 & 28.299 \\
\end{array}
\]

d. \( H_0: \sigma^2 = 225 \)
\( H_1: \sigma^2 < 225 \)

C. V. = 15.308 \hspace{1cm} d. f. = 28

2.

a. \( H_0: \sigma^2 = 225 \)
\( H_1: \sigma^2 > 225 \)

C. V. = 32.000 \hspace{1cm} d. f. = 16

b. \( H_0: \sigma^2 = 225 \)
\( H_1: \sigma^2 < 225 \)

C. V. = 8.907 \hspace{1cm} d. f. = 19

c. \( H_0: \sigma^2 = 225 \)
\( H_1: \sigma^2 \neq 225 \)

C. V. = 3.074, 28.299 \hspace{1cm} d. f. = 12

3.

a. \( 0.01 < P\text{-value} < 0.025 \) (0.015)

b. \( 0.005 < P\text{-value} < 0.01 \) (0.006)

c. \( 0.01 < P\text{-value} < 0.02 \) (0.012)

d. \( P\text{-value} < 0.005 \) (0.003)

4

a. \( 0.02 < P\text{-value} < 0.05 \) (0.037)

b. \( 0.05 < P\text{-value} < 0.10 \) (0.088)

c. \( 0.05 < P\text{-value} < 0.10 \) (0.066)

d. \( P\text{-value} < 0.01 \) (0.007)

5.

\( H_0: \sigma = 15 \)
\( H_1: \sigma < 15 \) (claim)

C. V. = 4.575 \hspace{1cm} \( \alpha = 0.05 \) \hspace{1cm} d. f. = 11

\( \chi^2 = \frac{(\sigma - \bar{\sigma})^2}{\sigma^2} = \frac{(12 - 13.6)^2}{15^2} = 9.0425 \)

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is less than 15.
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6.  
H₀:  \( \sigma^2 = 100 \)  
Hₜ:  \( \sigma^2 \neq 100 \)  (claim)  

\[ s^2 = 135.4333 \]  
C. V. = 2.700, 19.023  \( \alpha = 0.05 \)  d. f. = 9  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(135.4333)}{100} = 12.189 \]  

Do not reject the null hypothesis. There is not enough evidence to support the claim that the variance differs from 100.

7.  
H₀:  \( \sigma = 1.2 \)  (claim)  
Hₜ:  \( \sigma > 1.2 \)  

\( \alpha = 0.01 \)  d. f. = 14  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)(1.8)}{(1.2)^2} = 31.5 \]  

P-value < 0.005  (0.0047)  
Since P-value < 0.01, reject the null hypothesis. There is enough evidence to reject the claim that the standard deviation is less than or equal to 1.2 minutes.

8.  
H₀:  \( \sigma = 0.03 \)  (claim)  
Hₜ:  \( \sigma > 0.03 \)  

\[ s = 0.043 \]  
\( \alpha = 0.05 \)  d. f. = 7  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(7)(0.043)^2}{0.03^2} = 14.381 \]  

0.025 < P-value < 0.05  (0.045)  
Since P-value < 0.05, reject the null hypothesis. There is enough evidence to reject the claim that the standard deviation is less than or equal to 0.03 ounce.

9.  
H₀:  \( \sigma = 100 \)  
Hₜ:  \( \sigma > 100 \)  (claim)  

9. continued  
\( s = 126.721 \)  
C. V. = 12.017  \( \alpha = 0.10 \)  d. f. = 7  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(8-1)(126.721)^2}{100^2} = 11.241 \]  

0  \( \uparrow \) 12.017 11.241  

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is more than 100 mg.

10.  
H₀:  \( \sigma^2 = 100 \)  
Hₜ:  \( \sigma^2 > 100 \)  (claim)  

\[ s^2 = 183.7755 \]  
C. V. = 23.685  \( \alpha = 0.05 \)  d. f. = 14  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)(183.7755)}{100} = 25.729 \]  

0  \( \uparrow \) 23.685 25.729  

Reject the null hypothesis. There is enough evidence to support the claim that the variance is more than 100.

11.  
H₀:  \( \sigma = 35 \)  
Hₜ:  \( \sigma < 35 \)  (claim)  

C. V. = 3.940  \( \alpha = 0.05 \)  d. f. = 10  

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(11-1)(32)^2}{35^2} = 8.359 \]
11. continued

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is less than 35.

12.

\[ H_0: \sigma = 8 \]
\[ H_1: \sigma > 8 \] (claim)

C. V. = 55.758 \[ \alpha = 0.05 \] d. f. = 49

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(50-1)(10.5)^2}{8} = 84.410 \]

Reject the null hypothesis. There is enough evidence to support the claim that the standard deviation is more than 8.

13. continued

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is greater than 150 mg of sodium.

14.

\[ H_0: \sigma = 12 \]
\[ H_1: \sigma \neq 12 \] (claim)

\[ s = 14.608 \]
\[ C. V. = 5.226, 21.026 \]
\[ \alpha = 0.10 \]
\[ d. f. = 12 \]

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(13-1)(14.608)^2}{12} = 17.783 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation differs from 12 mg.

15.

\[ H_0: \sigma = 0.52 \]
\[ H_1: \sigma > 0.52 \] (claim)

\[ s = 162.09 \]
\[ C. V. = 21.026 \]
\[ \alpha = 0.05 \]
\[ d. f. = 12 \]

\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(0.568)^2}{(0.52)^2} = 22.670 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is more than 0.52 mm.

16.

\[ H_0: \sigma^2 = 9 \]
\[ H_1: \sigma^2 > 9 \] (claim)
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16. continued
\[ s^2 = 9.344 \]
C. V. = 14.684  \[ \alpha = 0.10 \]  d. f. = 9
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(9.344)}{9} = 9.344 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the variance is more than 9.

17. H₀: \( \sigma = 60 \) (claim)
H₁: \( \sigma \neq 60 \)
C. V. = 8.672, 27.587  \[ \alpha = 0.10 \]
d. f. = 17
\[ s = 64.6 \]
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(18-1)(64.6)^2}{60^2} = 19.707 \]

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the standard deviation is 60.

18. H₀: \( \sigma = 8 \)
H₁: \( \sigma > 8 \) (claim)
C. V. = 30.144  \[ \alpha = 0.05 \]  d. f. = 19
\[ X = 46.3 \]
\[ s = 11.017 \]
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(11.017)^2}{8^2} = 36.033 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is higher than 8 degrees.

19. \[ \sigma \approx \text{Range} \]
\[ \sigma \approx \frac{9400 - 6782}{4} = 679.50 \]
H₀: \( \sigma = 679.50 \)
H₁: \( \sigma \neq 679.50 \) (claim)
\[ s = 770.67 \]
C. V. = 5.009, 24.736  \[ \alpha = 0.05 \]  d. f. = 13
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(14-1)(770.67)^2}{679.50^2} = 16.723 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation differs from $679.50.

20. H₀: \( \sigma = 2385.9 \)
H₁: \( \sigma < 2385.9 \) (claim)
\[ s = 2194.845 \]
C. V. = 1.145  \[ \alpha = 0.05 \]  d. f. = 5
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(6-1)(2194.845)^2}{2385.9^2} = 4.231 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is less than 2385.9 feet.

EXERCISE SET 8-6

1. H₀: \( \mu = 25.2 \)
H₁: \( \mu \neq 25.2 \) (claim)
1. continued

C. V. = ± 2.032

\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{28.7 - 25.2}{4.6/\sqrt{35}} = 4.50 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average age differs from 25.2.

The 95% confidence interval of the mean is:

\[ X - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < X + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \]

\[ 28.7 - 2.032 \left( \frac{4.6}{\sqrt{35}} \right) < \mu < 28.7 + 2.032 \left( \frac{4.6}{\sqrt{35}} \right) \]

27.1 < \mu < 30.3

(TI: 27.2 < \mu < 30.2)

The confidence interval does not contain the hypothesized mean age of 25.2.

2.

H₀: \( \mu = \$236 \)
H₁: \( \mu \neq \$236 \) (claim)

C. V. = ± 2.539  d.f. = 19

\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{210 - 236}{43/\sqrt{4080}} = -2.704 \]

Reject the null hypothesis. There is enough evidence to support the claim that the average airfare differs from $236.

The 98% confidence interval of the mean is:

\[ X - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < X + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \]

\[ 210 - 2.539 \left( \frac{43}{\sqrt{4080}} \right) < \mu < 210 + 2.539 \left( \frac{43}{\sqrt{4080}} \right) \]

$185.59 < \mu < $234.41

There is agreement between the z-test and the confidence interval because the interval does not contain the mean of $236.

3.

H₀: \( \mu = \$19,150 \)
H₁: \( \mu \neq \$19,150 \) (claim)

C. V. = ± 1.96

\[ z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{17,020 - 19,150}{1.96} = -3.69 \]

Reject the null hypothesis. There is enough evidence to support the claim that the mean is not $19,150.

The 95% confidence interval supports the conclusion because it does not contain the hypothesized mean.

\[ X - 1.96 \frac{s}{\sqrt{n}} < \mu < X + 1.96 \frac{s}{\sqrt{n}} \]

$17,020 - 1.96 \left( \frac{4080}{\sqrt{50}} \right) < \mu < 17,020 + 1.96 \left( \frac{4080}{\sqrt{50}} \right) \]

$15,889 < \mu < $18,151

4.

H₀: \( p = 0.694 \)
H₁: \( p \neq 0.694 \) (claim)

C. V. = ± 1.96

\[ \hat{p} = \frac{120}{165} = 0.727 \]

\[ p = 0.694 \]

\[ q = 0.306 \]

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.727 - 0.694}{\sqrt{\frac{0.694 \cdot 0.306}{165}}} = 0.920 \]

Reject the null hypothesis. There is enough evidence to support the claim that the proportion differs from 0.694.
4. continued
The 95% confidence interval of the proportion is:
\[ \hat{p} - (z_{0.95}) \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + (z_{0.95}) \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]
\[ 0.727 - 1.96 \cdot \sqrt{\frac{0.273 \cdot 0.727}{155}} < p < 0.727 + 1.96 \cdot \sqrt{\frac{0.273 \cdot 0.727}{155}} \]
\[ 0.659 < p < 0.795 \]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the proportion of white collar criminals who serve time differs from 69.4%. The confidence interval does contain the hypothesized percentage of 69.4%.

5.
H₀: \( \mu = 19 \)
H₁: \( \mu \neq 19 \)  (claim)

C. V. = ± 2.145
\[ t = \frac{X - \mu}{s/\sqrt{n}} = \frac{21.3 - 19}{5.7/\sqrt{48}} = 1.37 \]

\[ -2.145 \quad 0 \quad \uparrow \quad 2.145 \quad 1.37 \]

The 99% confidence interval of the mean is:
\[ X - z_{0.99} \frac{\sigma}{\sqrt{n}} < \mu < X + z_{0.99} \frac{\sigma}{\sqrt{n}} \]
\[ 21.3 - 2.33 \cdot \frac{3}{\sqrt{48}} < \mu < 21.3 + 2.33 \cdot \frac{3}{\sqrt{48}} \]
\[ 17.7 < \mu < 24.9 \]

The decision is to reject the null hypothesis since 1.37 < 2.145 and the 99% confidence interval does not contain the hypothesized mean of 19. The conclusion is that there is enough evidence to reject the claim that the average time a person spends reading a newspaper is 10.8 minutes.

7. The power of a statistical test is the probability of rejecting the null hypothesis when it is false.

8. The power of a test is equal to \( 1 - \beta \) where \( \beta \) is the probability of a type II error.

9. The power of a test can be increased by increasing \( \alpha \) or selecting a larger sample size.

REVIEW EXERCISES - CHAPTER 8

1.
H₀: \( \mu = 18.3 \)
H₁: \( \mu \neq 18.3 \)  (claim)

C. V. = ± 2.33
\[ s = \sqrt{32.49} = 5.7 \]
\[ z = \frac{X - \mu}{s/\sqrt{n}} = \frac{20.9 - 18.3}{5.7/\sqrt{48}} = 3.16 \]
1. continued

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{23.9 - 25.3}{2.33/\sqrt{100}} = -2.19$

Reject the null hypothesis. There is enough evidence to support the claim that the average time spent online differs from 18.3 hours.

2. $H_0: \mu = 25.3$
$H_1: \mu < 25.3$ (claim)

$C. V. = -2.33$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average commute time is less than 25.3 minutes.

3. $H_0: \mu = 18,000$
$H_1: \mu < 18,000$ (claim)

$X = 16,298.37$ $s = 2604.82$

$C. V. = -2.33$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{16,298.37 - 18,000}{2604.82} = -3.58$

Reject the null hypothesis. There is enough evidence to support the claim that average debt is less than $18,000.

4. $H_0: \mu = 10$
$H_1: \mu < 10$ (claim)

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{2.33 - 10}{\sigma/\sqrt{n}} = -2.22$

P-value = 0.0132
Since 0.0132 < 0.05, reject the null hypothesis. The conclusion is that there is enough evidence to support the claim that the average time is less than 10 minutes.

5. $H_0: \mu = 1229$
$H_1: \mu \neq 1229$ (claim)

$C. V. = \pm 1.96$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1350 - 1229}{250/\sqrt{30}} = 1.875$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average rent differs from $1229.

6. $H_0: \mu = 50$
$H_1: \mu > 50$ (claim)

$C. V. = 1.356$ $d. f. = 12$

$X = 64.962$ $s = 51.929$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{64.962 - 50}{51.929} = 1.04$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the average rent differs from $50.
6. continued
Do not reject the null hypothesis. There is not enough evidence to support the claim that the average winnings exceed $50.

7.
H₀: μ = 10
H₁: μ ≠ 10 (claim)

\[ C. V. = -1.782 \quad \bar{X} = 9.6385 \quad s = 0.5853 \]
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{9.6385 - 10}{0.5853/\sqrt{13}} = -2.227 \text{ or } -2.230 \]

↑ - 1.782  0  - 2.230

Reject the null hypothesis. There is enough evidence to support the claim that average weight is less than 10 ounces.

8.
H₀: μ = 208
H₁: μ > 208 (claim)

\[ C. V. = 2.896 \quad \text{d. f.} = 8 \]
\[ \bar{X} = 209.74 \quad s = 1.67 \]
\[ t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{209.74 - 208}{1.67} = 3.13 \]

0  2.896  ↑  3.13

Reject the null hypothesis. There is enough evidence to support the claim that weight is greater than 208 grams.

9. continued
z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.28 - 0.17}{\sqrt{\frac{0.17(0.83)}{220}}} = 4.34

- 1.65  0  1.65  ↑  4.34

Reject the null hypothesis. There is enough evidence to support the claim that the percentage is greater than 17%.

10.
H₀: p = 0.602
H₁: p > 0.602 (claim)

\[ C. V. = 1.65 \]
\[ \bar{p} = 0.65 \quad p = 0.602 \quad q = 0.398 \]
\[ z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.65 - 0.602}{\sqrt{\frac{0.602(0.398)}{400}}} = 1.96 \]

0  1.65  ↑  1.96

Reject the null hypothesis. There is enough evidence to support the claim that the percentage of drug offenders is higher than 60.2%.

11.
H₀: p = 0.593
H₁: p < 0.593 (claim)

\[ C. V. = -2.33 \]
\[ \bar{p} = \frac{156}{300} = 0.52 \quad p = 0.593 \quad q = 0.407 \]
\[ z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.52 - 0.593}{\sqrt{\frac{0.593(0.407)}{300}}} = -2.57 \]

↑ - 2.33  0  - 2.57

159
11. continued
Reject the null hypothesis. There is enough evidence to support the claim that less than 59.3% of school lunches are free or at a reduced price.

12.
\( H_0: p = 0.65 \) (claim)
\( H_1: p \neq 0.65 \)

\[
\hat{p} = \frac{57}{80} = 0.7125 \quad p = 0.65 \quad q = 0.35
\]
\[
z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.7125 - 0.65}{\sqrt{\frac{(0.65)(0.35)}{80}}} = 1.17
\]
Area = 0.8790
P-value = 2(1 − 0.8790) = 0.242 (0.2412)
Since P-value > 0.05, do not reject the null hypothesis. There is not enough evidence to reject the claim that 65% of the teenagers own their own MP3 players.

13.
\( H_0: p = 0.204 \)
\( H_1: p \neq 0.204 \) (claim)

\[
\hat{p} = 0.18 \quad p = 0.204 \quad q = 0.796
\]
C. V. = ± 1.96
\[
z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.18 - 0.204}{\sqrt{\frac{(0.204)(0.796)}{300}}} = -1.03
\]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the proportion of high school smokers differs from 20.4%.

14. continued

\[
\begin{array}{c}
0 \quad 1.28 \quad \uparrow \\
3.03
\end{array}
\]

Reject the null hypothesis. There is enough evidence to support the claim that the percentage of men over the age of 65 still working is greater than 20.5%.

15.
\( H_0: \sigma = 4.3 \) (claim)
\( H_1: \sigma < 4.3 \)

d. f. = 19
\[
\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(2.6)^2}{(4.3)^2} = 6.95
\]
0.005 < P-value < 0.01 (0.006)
Since P-value < 0.05, reject the null hypothesis. There is enough evidence to reject the claim that the standard deviation is greater than or equal to 4.3 miles per gallon.

16.
\( H_0: \sigma^2 = 3.81 \)
\( H_1: \sigma^2 \neq 3.81 \) (claim)

\[
s^2 = (2.08)^2 = 4.3264
\]
C. V. = 5.629, 26.119 d. f. = 14
\[
\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)(4.3264)}{3.81} = 15.898
\]

\[
\begin{array}{c}
5.629 \quad \uparrow \\
15.898 \\
26.119
\end{array}
\]

Do not reject the null hypothesis. There is not enough evidence to support the claim that the variance of admission prices differs from 3.81.

17.
\( H_0: \sigma^2 = 40 \)
\( H_1: \sigma^2 \neq 40 \) (claim)
17. continued
\[ s = 6.6 \quad s^2 = (6.6)^2 = 43.56 \]
C. V. = 2.700, 19.023 \quad d. f. = 9
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(43.56)}{40} = 9.801 \]

\[ 2.700 \uparrow \]
9.801
19.023

Do not reject the null hypothesis. There is not enough evidence to support the claim that the variance differs from 40.

18. H₀: \( \sigma = 3.4 \) (claim)
H₁: \( \sigma \neq 3.4 \)
C. V. = 11.689, 38.076 \quad d. f. = 23
\[ \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(4.2)^2}{(3.4)^2} = 35.097 \]

\[ 11.689 \uparrow \]
38.076
35.097

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the standard deviation is 3.4 minutes.

19. continued
The decision is do not reject the null hypothesis since 1.49 < 2.58 and the confidence interval does contain the hypothesized mean of 4. There is not enough evidence to support the claim that the growth has changed. Yes, the results agree. The hypothesized mean is contained in the interval.

20. H₀: \( \mu = 35 \) (claim)
H₁: \( \mu \neq 35 \)
C. V. = ± 1.65
\[ z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{33.5 - 35}{\sqrt{36}} = -3.00 \]
The 90% confidence interval of the mean is:
\[ \bar{X} - z_{0.05} \frac{\sigma}{\sqrt{n}} < \bar{X} < \bar{X} + z_{0.05} \frac{\sigma}{\sqrt{n}} \]
\[ 33.5 - 1.65 \cdot \frac{3}{\sqrt{36}} < \mu < 33.5 + 1.65 \cdot \frac{3}{\sqrt{36}} \]
\[ 32.675 < \mu < 34.325 \]
The decision is to reject the null hypothesis since \(-3.00 < -1.65\) and the 90% confidence interval does not contain the hypothesized mean of 35. The conclusion is that there is enough evidence to reject the claim that the mean is 35 pounds.

CHAPTER 8 QUIZ

1. True
2. True
3. False, the critical value separates the critical region from the noncritical region.
4. True
5. False, it can be one-tailed or two-tailed.
6. b
7. d
8. c
9. b
10. Type I
11. \( \beta \)
12. Statistical hypothesis
13. Right
14. \( n - 1 \)
15. H₀: \( \mu = 28.6 \) (claim)
H₁: \( \mu \neq 28.6 \)
C. V. = ± 1.96
\[ z = 2.15 \]
15. continued
Reject the null hypothesis. There is enough evidence to reject the claim that the average age is 28.6.

16. \( H_0: \mu = \$6,500 \) (claim)
\( H_1: \mu \neq \$6,500 \)
C. V. = \( \pm 1.96 \)
z = 5.27
Reject the null hypothesis. There is enough evidence to reject the agent's claim.

17. \( H_0: \mu = 8 \)
\( H_1: \mu > 8 \) (claim)
C. V. = 1.65
z = 6.00
Reject the null hypothesis. There is enough evidence to support the claim that the average is greater than 8.

18. \( H_0: \mu = 500 \) (claim)
\( H_1: \mu \neq 500 \)
C. V. = \( \pm 3.707 \)
t = -0.571
Do not reject the null hypothesis. There is not enough evidence to reject the claim that the average is 500.

19. \( H_0: \mu = 67 \)
\( H_1: \mu < 67 \) (claim)
t = -3.1568
P-value < 0.005 (0.003)
Since P-value < 0.05, reject the null hypothesis. There is enough evidence to support the claim that the average height is less than 67 inches.

20. \( H_0: \mu = 12.4 \)
\( H_1: \mu < 12.4 \) (claim)
C. V. = -1.345
t = -2.324
Reject the null hypothesis. There is enough evidence to support the claim that the average is less than what the company claimed.

21. \( H_0: \mu = 63.5 \)
\( H_1: \mu > 63.5 \) (claim)
t = 0.47075
P-value > 0.25 (0.322)
Since P-value > 0.05, do not reject the null hypothesis. There is not enough evidence to support the claim that the average is greater than 63.5.

22. \( H_0: \mu = 26 \) (claim)
\( H_1: \mu \neq 26 \)
C. V. = \( \pm 2.492 \)
t = -1.5
Do not reject the null hypothesis. There is not enough evidence to reject the claim that the average is 26.

23. \( H_0: p = 0.39 \) (claim)
\( H_1: p \neq 0.39 \)
C. V. = \( \pm 1.96 \)
z = -0.62
Do not reject the null hypothesis. There is not enough evidence to reject the claim that 39% took supplements. The study supports the results of the previous study.

24. \( H_0: p = 0.55 \) (claim)
\( H_1: p < 0.55 \)
C. V. = -1.28
z = -0.8989
Do not reject the null hypothesis. There is not enough evidence to reject the survey's claim.

25. \( H_0: p = 0.35 \) (claim)
\( H_1: p \neq 0.35 \)
C. V. = \( \pm 2.33 \)
z = 0.666
Do not reject the null hypothesis. There is not enough evidence to reject the claim that the proportion is 35%.

26. \( H_0: p = 0.75 \) (claim)
\( H_1: p \neq 0.75 \)
C. V. = \( \pm 2.58 \)
z = 2.6833
Reject the null hypothesis. There is enough evidence to reject the claim.

27. The area corresponding to \( z = 2.15 \) is 0.9842.
P-value = 2(1 - 0.9842) = 0.0316

28. The area corresponding to \( z = 5.27 \) is greater than 0.9999. Thus, P-value \( \leq 2(1 - 0.9999) \leq 0.0002 \).
(TI: P-value < 0.0001)

29. \( H_0: \sigma = 6 \)
\( H_1: \sigma > 6 \) (claim)
C. V. = 36.415
\( \chi^2 = 54 \)
29. continued
Reject the null hypothesis. There is enough evidence to support the claim that the standard deviation is more than 6.

30. $H_0$: $\sigma = 8$ (claim)
$H_1$: $\sigma \neq 8$
C. V. = 27.991, 79.490
$\chi^2 = 33.2$
Do not reject the null hypothesis. There is not enough evidence to reject the claim that $\sigma = 8$.

31. $H_0$: $\sigma = 2.3$
$H_1$: $\sigma < 2.3$ (claim)
C. V. = 10.117
$\chi^2 = 13$
Do not reject the null hypothesis. There is not enough evidence to support the claim that the standard deviation is less than 2.3.

32. $H_0$: $\sigma = 9$ (claim)
$H_1$: $\sigma \neq 9$
$\chi^2 = 13.4$
P-value $> 0.20$ (0.291)
Since P-value $> 0.05$, do not reject the null hypothesis. There is not enough evidence to reject the claim that $\sigma = 9$.

33. $28.9 < \mu < 31.2$; no

34. $6562.81 < \mu < 6637.19$; no