Chapter 4: Probability and Counting Rules

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Section 4.1: Sample Spaces and Probability

Probability is the measure of the likeliness that an event will occur.

A probability experiment is a chance process that leads to well-defined results called outcomes.

An outcome is the result of a single trial of a probability experiment.

A sample space is a set of all possible outcomes of an experiment.

Exercise 1. An experiment consists of rolling a single die. Find the sample space.

Exercise 2. An experiment consists of rolling two dice. Find the sample space.
Exercise 3. Find the sample space for drawing one card from an ordinary deck of cards.

A **tree diagram** is a device consisting of line segments emanating from a starting point and also from the outcome point. It is used to determine all possible outcomes of a probability experiment.

Exercise 4. An experiment consist of flipping a coin twice. Find the sample space.

Exercise 5. Use a tree diagram to find the sample space for the gender of three children in a family.
Definition 1. An event is a subset of a sample space of an experiment.

Exercise 6. Suppose an experiment consists of tossing a coin three times and observing the sequence of heads and tails. Determine the event $E = \text{“exactly two heads.”}$

Exercise 7. Suppose that we have two urns - call them urn I and urn II - each containing red balls and white balls. An experiment consists of selecting an urn and then selecting a ball from that urn and noting its color.

a) What is a suitable sample space for this experiment?

b) Describe the event “urn I is selected” as a subset of the sample space.

There are three basic interpretations of probability:

1. Classical probability
2. Empirical or relative frequency probability
3. Subjective probability

Classical Probability uses sample spaces to determine the numerical probability that an event will happen.

Definition 2. Experiments in which each outcome has the same probability are said to be experiments with equally likely outcomes.
Definition 3. If an experiment with sample space $S$ has equally likely outcomes, then for any event $E$ the probability of $E$ is given by

$$P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ and $n(S)$ denote the number of elements in $E$ and $S$, respectively.

Note: Probability is always a number from 0 to 1. Impossible events always have probability 0 and certain events have probability 1. The sum of the probabilities of all the outcomes in a sample space is 1.

Exercise 8. Roll a single die. What is the probability that it lands on an odd number?

Exercise 9. Find the probability of getting a red face card (jack, queen, or king) when randomly drawing a card from an ordinary deck.

Exercise 10. If a family has three children, find the probability that exactly two of the three children are girls.
Note: In probability theory, it is important to understand the meaning of the words **and** and **or**. For example, if you were asked to find the probability of getting a queen **and** a heart when you were drawing a single card from a deck, you would be looking for the queen of hearts. Here the word **and** means “at the same time.” The word **or** has two meanings. There is an **inclusive or** and an **exclusive or**.

\[
P(a \text{ queen or a king}) = \\
P(a \text{ queen or a heart}) = \\
\]

**Exercise 11.** a) A king

b) The 4 of spades

c) A face card (jack, queen, or king)

d) A red card

e) A club

**Complement Rule:**

\[
P(E) = 1 - P(\overline{E}) \quad \text{and} \quad P(E) = 1 - P(\overline{E}) \quad \text{and} \quad P(E) + P(\overline{E}) = 1
\]
**Exercise 12.** In a study, it was found that 23% of the people surveyed said that vanilla was their favorite flavor of ice cream. If a person is selected at random, find the probability that the person’s favorite flavor of ice cream is not vanilla.

**Empirical Probability** is the type of probability that uses frequency distributions based on observations to determine numerical probabilities of events.

Given a frequency distribution, the probability of an event being in a given class is

\[ P(E) = \frac{\text{frequency for the class}}{\text{total frequencies in the distribution}} = \frac{f}{n} \]

**Exercise 13.** A researcher for the American Automobile Association (AAA) asked 50 people who plan to travel over the Thanksgiving holiday how they will get to their destination. The results can be categorized in a frequency distribution as shown. Find the probability that a person will travel by airplane over the Thanksgiving holiday.

<table>
<thead>
<tr>
<th>Method</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive</td>
<td>41</td>
</tr>
<tr>
<td>Fly</td>
<td>6</td>
</tr>
<tr>
<td>Train or bus</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
</tr>
</tbody>
</table>

**Exercise 14.** In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a) A person has type O blood.

b) A person has type A or type B blood.
c) A person has neither type A nor type O blood.

d) A person does not have type AB blood.

Exercise 15. Hospital records indicated that knee replacement patients stayed in the hospital for the number of days shown in the distribution.

<table>
<thead>
<tr>
<th>Number of days stayed</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>127</td>
</tr>
</tbody>
</table>

Find these probabilities.

a) A patient stayed exactly 5 days.

b) A patient stayed fewer than 6 days.

c) A patient stayed at most 4 days.
d) A patient stayed at least 5 days.

**Note:** Read in the textbook about Law of Large Numbers, Subjective Probability, and Probability and Risk Taking.