Chapter 4: Probability and Counting Rules

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Section 4.2: The Addition Rules for Probability

Definition 1. Two events are mutually exclusive or disjoint if they cannot both occur at the same time. That is, the events have no outcomes in common.

Exercise 1. Determine whether the two events are mutually exclusive. Explain your answer.
   a) Randomly selecting a female student. Randomly selecting a student who is a junior.
      \[ \text{not mut. excl.} \]
   b) Randomly selecting a person with type A blood. Randomly selecting a person with type O blood.
      \[ \text{mut. excl.} \]
   c) Rolling a die and getting an odd number. Rolling a die and getting a number less than 3.
      \[ \text{not mut. excl.} \]
   d) Randomly selecting a person who is under 21 years of age. Randomly selecting a person who is over 30 years of age.
      \[ \text{mut. excl.} \]

Exercise 2. Determine which events are mutually exclusive and which events are not when a single card is drawn from a deck.
   a) Getting a king; getting a diamond
      \[ \text{no} \]
   b) Getting a 4; getting a king
      \[ \text{yes} \]
   c) Getting a face card; getting a club
      \[ \text{no} \]

Definition 2. When two events A and B are mutually exclusive, the probability that A or B will occur is
\[ P(A \text{ or } B) = P(A) + P(B) \]
Exercise 3. A city has 9 coffee shops: 3 Starbucks, 2 Caribou Coffees, and 4 Crazy Mocho Coffees. If a person selects one shop at random to buy a cup of coffee, find the probability that it is either a Starbucks or Crazy Mocho Coffees.

\[ P(S \text{ or } CM) = \frac{3}{9} + \frac{4}{9} = \frac{7}{9} \approx 0.778 \]

Exercise 4. In a survey, 8% of the respondents said that their favorite ice cream flavor is cookies and cream, and 6% like mint chocolate chip. If a person is selected at random, find the probability that her or his favorite ice cream flavor is either cookies and cream or mint chocolate chip.

\[ P(CC \text{ or } MCC) = 0.08 + 0.06 = 0.14 \]

Definition 3. When two events A and B are not mutually exclusive, the probability that A or B will occur is

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Exercise 5. A single card is drawn at random from an ordinary deck of cards. Find the probability that it is either an ace or a black card.

Exercise 6. In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

<table>
<thead>
<tr>
<th>Nurses</th>
<th>Females</th>
<th>Males</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Physicians</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

\[ P(\text{nurse or male}) = \frac{10}{18} \approx 0.769 \]
Exercise 7. On New Year's Eve, the probability of a person driving while intoxicated is 0.32, the probability of a person having a driving accident is 0.09, and the probability of a person having a driving accident while intoxicated is 0.06. What is the probability of a person driving while intoxicated or having a driving accident?

\[
P(\text{intoxicated or accident}) = P(\text{intox}) + P(\text{accident}) - P(\text{intox and accident})
\]

\[
= 0.32 + 0.09 - 0.06 = 0.35
\]

Note: For three events that are not mutually exclusive,

\[
P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C)
\]

Exercise 8. Assume that following an injury you received from playing your favorite sport, you obtain and read information on new pain medications. In that information you read of a study that was conducted to test the side effects of two new pain medications. Use the following table to answer the questions and decide which, if any, of the two new pain medications you will use.

<table>
<thead>
<tr>
<th>Side effect</th>
<th>Placebo</th>
<th>Drug A</th>
<th>Drug B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n=192</td>
<td>n=186</td>
<td>n=186</td>
</tr>
<tr>
<td>Upper respiratory</td>
<td>10</td>
<td>32</td>
<td>19</td>
</tr>
<tr>
<td>congestion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sinus headache</td>
<td>11</td>
<td>25</td>
<td>32</td>
</tr>
<tr>
<td>Stomach ache</td>
<td>2</td>
<td>48</td>
<td>12</td>
</tr>
<tr>
<td>Neurological headache</td>
<td>34</td>
<td>55</td>
<td>72</td>
</tr>
<tr>
<td>Cough</td>
<td>22</td>
<td>18</td>
<td>31</td>
</tr>
<tr>
<td>Lower respiratory</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>congestion</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) How many subjects were in the study? \(192 + 186 + 186 = 566\)

b) How long was the study? \(12 \text{ weeks}\)

c) What were the variables under study?

\(\text{type of pain reliever and side effects}\)
d) What type of variables are they, and what level of measurement are they on?

both var. are qualitative and nominal

e) What is the probability that a randomly selected person was receiving a placebo?

\[
\frac{192}{566} = 0.339
\]

f) What is the probability that a person was receiving a placebo or drug A? Are these mutually exclusive events? What is the complement to this event?

\[
\frac{192 + 126}{566} = 0.668
\]

meet. excl.

randomly selected person was receiving drug B

g) What is the probability that a randomly selected person was receiving a placebo or experienced a neurological headache?

\[
\frac{192 + 55 + 72}{566} = 0.564
\]

h) What is the probability that a randomly selected person was not receiving a placebo or experienced a sinus headache?

\[
\frac{(186 + 188) + (11 + 25 + 32) - (25 + 32)}{566} = 0.680
\]