Chapter 4: Probability and Counting Rules

Diana Pell

Section 4.3: The Multiplication Rules and Conditional Probability

Two events $A$ and $B$ are independent events if the fact that $A$ occurs does not affect the probability of $B$ occurring.

Some examples of independent events:

1) Rolling a die and getting a 6, and then rolling a second die and getting a 3.

2) Drawing a card from a deck and getting a queen, replacing it, and drawing a second card and getting a queen.

Multiplication Rule 1

When two events $A$ and $B$ are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Exercise 1. A coin is flipped and a die is rolled. Find the probability of getting heads on the coin and a 4 on the die.

Exercise 2. A card is drawn from a deck and replaced; then a second card is drawn. Find the probability of getting a queen and then an ace.
Exercise 3. An urn contains 3 red balls, 2 green balls, and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a) Selecting 2 green balls

b) Selecting 1 green ball and then 1 white ball

c) Selecting 1 red ball and then 1 green ball

\[ \frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100} \]

Exercise 4. A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.

Exercise 5. Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.
When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be dependent events.

Some examples of dependent events:

1) Drawing a card from a deck, not replacing it, and then drawing a second card
2) Selecting a ball from an urn, not replacing it, and then selecting a second ball
3) Having high grades and getting a scholarship
4) Parking in a no-parking zone and getting a parking ticket

Conditional Probability

A card is drawn from a deck of 52 cards. Given that the card drawn was a face card, what is the prob. that it was red?

\[ P(\text{red} | \text{face card}) = \frac{6}{12} = \frac{1}{2} \]

Definition 1. The probability that a second event B occurs given that a first event A has occurred can be found by dividing the probability that both events occurred by the probability that the first event has occurred. The formula is

\[ P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \]

Exercise 7. Suppose that your professor goes stark raving mad and chooses your final grade from A, B, C, D, F, or Incomplete totally at random. Find the probability of getting an A given that you get a letter grade higher than D.

\[ P(A | > D) = \frac{1}{3} \]
Exercise 8 (You Try!) A group of patients in a blind drug trial is assigned numbers from 1 through 8. The even numbers get an experimental drug, while the odd numbers get a placebo. If Eleanor is one of the patients, what’s the probability that she’s getting the experimental drug given that she wasn’t assigned 1, 2, or 3?

\[ P(\text{exp.} \mid \text{not 1, 2 or 3}) = \frac{3}{5} \]

Exercise 8 The probability that Sam parks in a no-parking zone and gets a parking ticket is 0.06, and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.20. On Tuesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

\[ P(\text{ticket} \mid \text{no-parking}) = \frac{0.06}{0.20} = \frac{6}{20} = \frac{3}{10} = 0.3 \]

Exercise 8 A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>32</td>
<td>18</td>
<td>50</td>
</tr>
<tr>
<td>Female</td>
<td>8</td>
<td>42</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>60</td>
<td>100</td>
</tr>
</tbody>
</table>

Find these probabilities.

a) The respondent answered yes, given that the respondent was a female.

\[ P(\text{yes} \mid \text{female}) = \frac{8}{50} = 0.16 \]
b) The respondent was a male, given that the respondent answered no.

\[ P(\text{male} \mid \text{no}) = \frac{18}{60} = 0.30 \]

**Multiplication Rule 2**

When two events are dependent, the probability of both occurring is

\[ P(A \text{ and } B) = P(A) \cdot P(B \mid A) \]

**Exercise 10.** In a recent survey, 33% of the respondents said that they feel that they are overqualified (O) for their present job. Of these, 24% said that they were looking for a new job (J). If a person is selected at random, find the probability that the person feels that he or she is overqualified and is also looking for a new job.

\[ P(O \text{ and } J) = P(O) \cdot P(J \mid O) = (0.33)(0.24) \approx 0.0792 \]

**Exercise 11.** World Wide Insurance Company found that 53% of the residents of a city had homeowner's insurance (H) with the company. Of these clients, 27% also had automobile insurance (A) with the company. If a resident is selected at random, find the probability that the resident has both homeowner's and automobile insurance with World Wide Insurance Company.

\[ P(H \text{ and } A) = P(H) \cdot P(A \mid H) = (0.53)(0.27) \approx 0.1421 \]

**Exercise 12.** Three cards are drawn from an ordinary deck and not replaced. Find the probability of these events.

a) Getting 3 jacks

\[ P(3 \text{ jacks}) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \approx 0.0002 \]
b) Getting an ace, a king, and a queen in order

\[ P(\text{Ace and } K \text{ and } Q) = \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \approx 0.0008 \]

\[ \boxed{0.0008} \]

c) Getting a club, a spade, and a heart in order

\[ P(\text{Clubs and Spades and Hearts}) = \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{13}{50} \approx 0.0171 \]

\[ \boxed{0.0171} \]

d) Getting 3 clubs

\[ P(3 \text{ Clubs}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \approx 0.013 \]

\[ \boxed{0.013} \]

**Exercise 18**

Box 1 contains 2 red balls and 1 blue ball. Box 2 contains 3 blue balls and 1 red ball. A coin is tossed. If it falls heads up, box 1 is selected and a ball is drawn. If it falls tails up, box 2 is selected and a ball is drawn. Find the probability of selecting a red ball.

\[ P(\text{red}) = \]

\[ \frac{1}{2} \frac{3}{4} \frac{3}{4} \]

\[ \frac{1}{2} \frac{3}{4} \frac{3}{4} \]

\[ \frac{1}{2} \frac{3}{4} \frac{3}{4} \]

\[ \frac{1}{2} \frac{3}{4} \frac{3}{4} \]
Probabilities for "At Least"

**Exercise 14.** A person selects 3 cards from an ordinary deck and replaces each card after it is drawn. Find the probability that the person will get at least one heart.

\[
P(\text{at least one heart}) = 1 - P(\text{no hearts})
\]

\[
= 1 - \frac{29}{52} \cdot \frac{29}{52} \cdot \frac{29}{52} \approx 0.578
\]

**Exercise 15.** A coin is tossed 5 times. Find the probability of getting at least 1 tail.

\[
P(\text{at least one tail}) = 1 - P(\text{all heads})
\]

\[
= 1 - \left( \frac{1}{2} \right)^5 = \frac{31}{32} \approx 0.969
\]

**Exercise 16.** The Neckware Association of America reported that 3% of ties sold in the United States are bow ties. If 4 customers who purchased a tie are randomly selected, find the probability that at least 1 purchased a bow tie.

\[
P(\text{at least one purchased a bow tie}) = 1 - P(\text{no bow ties one purchased})
\]

\[
= 1 - (.97)^4 \cdot (.97)^4 \cdot (.97)^4 \approx .115
\]

There is a 11.5% chance of a person purchasing at least one bow tie.