Chapter 11: Probability and Counting Techniques

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Section 11.1: The Fundamental Counting Principle

Exercise 1. How many different two-letter words (including nonsense words) can be formed when repetition of letters is allowed? When repetition is NOT allowed?

Exercise 2. How many license plates consisting of two letters followed by four digits are possible?

Exercise 3. How many license plates consisting of two letters followed by four digits are possible if letters cannot be repeated?

Exercise 4. How many license plates consisting of two letters followed by four digits are possible if digits cannot be related?
Exercise 5. How many license plates consisting of two letters followed by four digits are possible if both letters and digits cannot be repeated?

Exercise 6. How many ways can five people be arranged in a line for a group picture?

Exercise 7. Toss a coin six times and observe the sequence of heads or tails that results. How many different sequences are possible?

Exercise 8. Six houses in a row are each to be painted with one of the colors red, blue, green, and yellow. In how many different ways can the houses be painted so that no two adjacent houses are of the same color?

Exercise 9. How many three-digit odd numbers can be formed using the digits 1, 2, 3, 4, 5, 6, and 7?

Sections 11.1 - 11.2: Combinations and Permutations

A construction crew has three members. A team of two must be chosen for a particular job. In how many ways can the team be chosen?
How many words (strings of letters) of two distinct letters can be formed from the letters \{a, b, c\}.

Factorial

**Combinations** (Order does not matter!)

**Combination Formula:** The number of combinations of \(n\) objects taken \(r\) at a time is

\[
C(n, r) = \frac{n!}{(n - r)!r!}
\]

**Exercise 10.** a) \(C(100, 2)\)

b) \(C(5, 5)\)
Exercise 11. The board of directors of a corporation has 10 members. In how many ways can they choose a committee of 3 board members to negotiate a merger?

Exercise 12. How many different selections of two books can be made from a set of nine books?

Permutations (Order matters!)

Permutation Formula: The number of permutations of $n$ objects taken $r$ at a time is

$$P(n, r) = n(n - 1)(n - 2)(n - 3) \cdots (n - r + 1)$$

OR

$$P(n, r) = \frac{n!}{(n - r)!}$$

Exercise 13. Eight horses are entered in a race in which a first, second, and third prize will be awarded. Assuming no ties, how many different outcomes are possible?
Exercise 14. A club has 10 members. In how many ways can they choose a slate of four officers, consisting of a president, vice president, secretary, and treasurer?

Exercise 15. How many ways can you arrange 5 of 10 books on a shelf?

Exercise 16. Suppose that you own 10 sweaters and are going on a trip. How many ways can you select six of them to leave at home?

Exercise 17. Of the 20 applicants for a job, 4 will be selected for intensive interviews. In how many ways can the selection be made?

Exercise 18. A poker hand consists of 5 cards selected from a deck of 52 cards.
a) How many different poker hands are there?

b) How many different poker hands consist entirely of aces and kings?

c) How many different poker hands consist entirely of clubs?

d) How many different poker hands consist entirely of red cards?

Exercise 19. In how many ways can five mathematics books and four novels be placed on a bookshelf if the mathematics books must stay together?

Solving a Permutation Problem with Like Objects
Exercise 20. How many different passwords can be made using all the letters in the word *Mississippi*?

Exercise 21. (You Try!) How many different passwords can be made using all the letters in the word *Massachusetts*?

Exercise 22. A committee has four male and five female members. In how many ways can a subcommittee consisting of two males and two females be selected?
Exercise 23. An urn contains 25 numbered balls, of which 15 are red and 10 are white. A sample of 3 balls is to be selected.

a) How many different samples are possible?

b) How many samples contain all red balls?

c) How many samples contain 1 red balls and 2 white balls?

d) How many samples contain at least 2 red balls?
Exercise 24. (You Try!) A four-person crew for the international space station is to be chosen from a candidate pool of 10 Americans and 12 Russians. How many different crews are possible if there must be at least two Russians?