Chapter 8: Hypothesis Testing

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Section 8.1 and 8.3: Hypothesis Testing and $t$ Test for a Mean

A statistical hypothesis is a claim or statement about a property of a population.

A hypothesis test (or significance test) is a procedure for testing a claim about a property of a population.

The null hypothesis, symbolized by $H_0$, is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is equal to some claimed value. (The term null is used to indicate no change.)

The alternative hypothesis, symbolized by $H_1$, is a statement that the parameter has a value that somehow differs from the null hypothesis. The symbolic form of the alternative hypothesis must use one of these symbols: $<, >, \neq$.

Situation A

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

$H_0 : \mu = 82 \quad \text{and} \quad H_1 : \mu \neq 82$

(null hyp) (alt. hyp.)

This test is called a two-tailed test: the critical region is in the two extreme regions (tails) under the curve.

Situation B

A chemist invents an additive to increase the life of an automobile battery. The mean lifetime of battery is 36 months.

$H_0 : \mu = 36 \quad \text{and} \quad H_1 : \mu > 36$

This test is called right-tailed: the critical region is in the extreme right region (tail) under the curve.
Situation C

A contractor wishes to lower heating bills by using a special type of insulation in houses.

\[ \text{Ave. monthly heating bill is } \$78. \]

\[ H_0: \mu = 78 \quad \text{and} \quad H_1: \mu < 78 \]

This test is left-tailed test: the critical region is in the extreme left region (tail) under the curve.

<table>
<thead>
<tr>
<th>Two-tailed test</th>
<th>Right-tailed test</th>
<th>Left-tailed test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \mu = k )</td>
<td>( H_0: \mu = k )</td>
<td>( H_0: \mu = k )</td>
</tr>
<tr>
<td>( H_1: \mu \neq k )</td>
<td>( H_1: \mu &gt; k )</td>
<td>( H_1: \mu &lt; k )</td>
</tr>
</tbody>
</table>

Note: In this book, the null hypothesis is always stated using the equals sign.

Exercise 1. State the null and alternative hypotheses for each conjecture.

a) A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.

\[ H_0: \mu = 8.6 \quad \text{and} \quad H_1: \mu > 8.6 \]

b) An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of USB drives by using robots instead of humans for certain tasks. The mean number of defective drives per 1000 is 18.

\[ H_0: \mu = 18 \quad \text{and} \quad H_1: \mu < 18 \]

c) A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

\[ H_0: \mu = 73 \quad \text{and} \quad H_1: \mu \neq 73 \]
The critical or rejection region is the range of test values that indicates that there is a significant difference and that the null hypothesis should be rejected.

The noncritical or nonrejection region is the range of test values that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

The critical value separates the critical region from the noncritical region. The symbol for critical value is C.V.

A one-tailed test indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean. A one-tailed test is either a right-tailed test or a left-tailed test, depending on the direction of the inequality of the alternative hypothesis.

In a two-tailed test the null hypothesis should be rejected when the test value is in either of the two critical regions.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Claim is $H_0$</th>
<th>Claim is $H_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>There is enough evidence to reject the claim.</td>
<td>There is enough evidence to support the claim.</td>
</tr>
<tr>
<td>Do not reject $H_0$</td>
<td>There is not enough evidence to reject the claim.</td>
<td>There is not enough evidence to support the claim.</td>
</tr>
</tbody>
</table>

The $t$ test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed and $\sigma$ is unknown.

The formula for the $t$ test is

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

$$d.f = n - 1$$
Assumptions for the t Test for a Mean When $\sigma$ is Unknown

1. The sample is a random sample.
2. Either $n \geq 30$ or the population is normally distributed when $n < 30$.

When you test hypotheses by using the $t$ test (traditional method), follow the following procedure:

1. State the hypotheses and identify the claim.
2. Find the critical value(s) from Table F.
3. Compute the test value.
4. Make the decision to reject or not reject the null hypothesis.
5. Summarize the results.

Note: Remember that the $t$ test should be used when the population is approximately normally distributed and the population standard deviation is unknown.

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Exercise 2. Find the critical $t$ value for $\alpha = 0.05$ with d.f. = 16 for right-tailed $t$ test.

\[ t_{\alpha = 0.05} \text{ d.f. = 16} \]

\[ CV \approx 1.746 \]

Exercise 3. Find the critical $t$ value for $\alpha = 0.01$ with d.f. = 22 for left-tailed $t$ test.

\[ CV \approx -2.508 \]

Exercise 4. Find the critical $t$ value for $\alpha = 0.10$ with d.f. = 18 for two-tailed $t$ test.

\[ CV = 1.734 \]
\[ CV = -1.734 \]

Exercise 5. Find the critical $t$ value for $\alpha = 0.05$ with d.f. = 28 for right-tailed $t$ test.

\[ CV = 1.701 \]
Exercise 6. A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at \( \alpha = 0.05 \)? Assume the variable is normally distributed.

1. \( H_0 : \mu = 16.3 \) (claim) and \( H_1 : \mu \neq 16.3 \)

2. Ci. Values: 2.262 and -2.262 for \( \alpha = 0.05 \) and d.f. = 10 - 1 = 9

3. \[ t = \frac{\bar{X} - \mu}{s} = \frac{17.7 - 16.3}{1.8} = 2.460 \]

4. Reject the null hypothesis since 2.460 > 2.262.

5. There's enough evidence to reject the claim that the average number of infections is 16.3.

Exercise 7. According to payscale.com, the average starting salary for a nurse practitioner is $79,500. A researcher wishes to test the claim that the starting salary is less than $79,500. A random sample of 8 starting nurse practitioners is selected, and their starting salaries (in dollars) are shown. Is there enough evidence to support the researcher's claim at \( \alpha = 0.10 \)? Assume the variable is normally distributed.

1. \( H_0 : \mu = 79,500 \) and \( H_1 : \mu < 79,500 \) (claim)

2. \( \alpha = 0.10 \) and d.f. = 8 - 1 = 7 \( \rightarrow \) Ci. = -1.415

3. \[ \bar{X} = \$75,150, \quad s = \$6,937.68 \]

4. \[ t = \frac{\bar{X} - \mu}{s} = \frac{75,150 - 79,500}{\frac{6,937.68}{\sqrt{8}}} = -1.773 \]

5. Reject the null hyp. since -1.773 falls in the critical region.

6. There's enough evidence to support the claim that the average starting salary for nurse practitioners is less than $79,500.
Exercise 8. The average cost for teeth straightening with metal braces is approximately $5400. A nationwide franchise thinks that its cost is below that figure. A random sample of 28 patients across the country had an average cost of $5250 with a standard deviation of $629. At $\alpha = 0.025$, can it be concluded that the mean is less than $5400? 

a) State the hypotheses and identify the claim. 

$$H_0: \mu = 5400 \quad \text{and} \quad H_1: \mu < 5400 \quad \text{(claim)}$$

b) Find the critical value(s). 

$$C.V. = -2.052$$ 

$$d.f. = 27$$

c) Find the test value. 

$$t = \frac{5250 - 5400}{\frac{629}{\sqrt{28}}} = -1.262$$

d) Make the decision. 

Do not reject the null hypothesis.

e) Summarize the results. 

There is not enough evidence to support the claim that the average cost is less than $5400.
Exercise 9. The National Novel Writing Association states that the average novel is at least 50,000 words. A particularly ambitious writing club at a college-preparatory high school had randomly selected members with works of the following lengths. At \( \alpha = 0.10 \), is there sufficient evidence to conclude that the mean length is greater than 50,000 words?

\[
\begin{align*}
48,972 & \quad 50,100 & \quad 51,560 \\
49,800 & \quad 50,020 & \quad 49,900 \\
52,193 & \\
\end{align*}
\]

a) State the hypotheses and identify the claim.

\[ H_0 : \mu = 50,000 \quad \text{and} \quad H_1 : \mu > 50,000 \quad (\text{claim}) \]

b) Find the critical value(s).

\[ C.V. = 1.440 \]

\[ d.f. = 6 \]

c) Find the test value.

\[ X = 50,363, s = 1113.16 \]

\[ t = \frac{50,363 - 50,000}{1113.16} \]

\[ t = 0.869 \]

d) Make the decision.

Do not reject the null hypothesis.

e) Summarize the results.

There's not enough evidence to support the claim that the average number of words is greater than 50,000.
Review from Chapter 7

**Exercise 10.** A U.S. Travel Data Center survey reported that Americans stayed an average of 7.5 nights when they went on vacation. The sample size was 1500. Find a point estimate of the population mean. Find the 95% confidence interval of the true mean. Assume the population standard deviation was 0.8.

\[
\bar{X} = 7.5 \text{ is the point estimate of } \mu.
\]

\[
\bar{X} - 2\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 2\frac{\sigma}{\sqrt{n}}
\]

\[
7.5 - 1.96 \left(\frac{0.8}{\sqrt{1500}}\right) < \mu < 7.5 + 1.96 \left(\frac{0.8}{\sqrt{1500}}\right)
\]

\[
7.46 < \mu < 7.54
\]

**Exercise 11.** In a survey of 1004 randomly selected individuals, 442 felt that President George W. Bush spent too much time away from Washington. Find a 95% confidence interval for the true population proportion.

\[
\hat{p} - 2\frac{\hat{p}\hat{q}}{n} < \hat{p} < \hat{p} + 2\frac{\hat{p}\hat{q}}{n}
\]

\[
0.44 - 1.96 \sqrt{\frac{(0.44)(0.56)}{1004}} < \hat{p} < 0.44 + 1.96 \sqrt{\frac{(0.44)(0.56)}{1004}}
\]

\[
0.409 < \hat{p} < 0.471
\]

**Exercise 12.** A random sample of 22 lawn mowers was selected, and the motors were tested to see how many miles per gallon of gasoline each one obtained. The variance of the measurements was 2.6. Find the 95% confidence interval of the true variance. Assume the variable is normally distributed.

\[
\frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(\alpha/2)}}
\]

\[
\frac{(22-1)(2.6)}{35.979} < \sigma^2 < \frac{(22-1)(2.6)}{10.288}
\]

\[
1.5 < \sigma^2 < 5.8
\]

We can be 95% confident that true variance for the # of miles per gallon of gasoline each one obtained is between 1.5 and 5.8.
<table>
<thead>
<tr>
<th>d.f.</th>
<th>One tail, α</th>
<th>Two tails, α</th>
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<tbody>
<tr>
<td>1</td>
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<td>3.314</td>
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<tr>
<td>2</td>
<td>1.886</td>
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<td>3</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>1.476</td>
<td>2.115</td>
</tr>
<tr>
<td>6</td>
<td>1.440</td>
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<tr>
<td>7</td>
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<td>9</td>
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<td>1.439</td>
</tr>
<tr>
<td>1000</td>
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</tr>
</tbody>
</table>

* (z) = 1.282 \times 15.689 

\* 1.960 

\* 2.330 

\* 2.581 

\* 2.779